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OPTIMIZATION OF MOBILE ROBOT MOVEMENT ON A PLANE WITH FINITE NUMBER OF REPELLER SOURCES

Kostjukov V., Medvedev M., Pshikhopov V. **Optimization of mobile robot movement on a plane with finite number of repeller sources.**

Abstract. The paper considers the problem of planning a mobile robot movement in a conflict environment, which is characterized by the presence of areas that impede the robot to complete the tasks. The main results of path planning in the conflict environment are considered. Special attention is paid to the approaches based on the risk functions and probabilistic methods. The conflict areas, which are formed by point sources that create in the general case asymmetric fields of a continuous type, are observed. A probabilistic description of such fields is proposed, examples of which are the probability of detection or defeat of a mobile robot. As a field description, the concept of characteristic probability function of the source is introduced; which allows us to optimize the movement of the robot in the conflict environment. The connection between the characteristic probability function of the source and the risk function, which can be used to formulate and solve simplified optimization problems, is demonstrated. The algorithm for mobile robot path planning that ensures the given probability of passing the conflict environment is being developed. An upper bound for the probability of the given environment passing under fixed boundary conditions is obtained. A procedure for optimizing the robot path in the conflict environment is proposed, which is characterized by higher computational efficiency achieved by avoiding the search for an exact optimal solution to a suboptimal one. A procedure is proposed for optimizing the robot path in the conflict environment, which is characterized by higher computational efficiency achieved by avoiding the search for an exact optimal solution to a suboptimal one. The proposed algorithms are implemented in the form of a software simulator for a group of ground-based robots and are studied by numerical simulation methods.

Keywords: Path Planning, Conflict Environment, Movement Optimization, Characteristic Probability Function.

1. Introduction. The task of a mobile robot path planning is one of the most relevant problems of modern robotic science [1-5]. When planning paths in the conflict environment it is necessary not only to take into account the local obstacles, but also the presence of areas that impede the mobile robot from completing its tasks. The sources of such areas can be detecting equipment, sources of radioactive or chemical pollution, etc. This raises the problem of avoiding the effect of such sources by the mobile robot [6]. The popular approach to solving this problem in uncertain conditions is the use of AI technologies [7-9] and methods that utilize the probability theory or risk functions [10 –18].

Thus, in the papers [7-9] the different aspects of the neural network use related to the planning of movement tasks in the conflict environments are considered. P. Agrawal and H. Agrawal [7] solve the problem of the evasive objects group persecution. Hunters have to recognize the friendly robot groups and perform the path planning to persecute the evasive objects.

Developed neural network solves the persecution problem taking into account the delays and limitations of communication channels. The network predicts the evasive objects' motion. The proposed approach advantages in an obstructed environment are demonstrated by simulations. The paper [8] proposes a biologically inspired neural algorithm based on the dynamic threat assessment. The algorithm is a combination of a fuzzy set, neural network, artificial potential field, and Bellman optimization procedure. The effectiveness of the presented algorithm is demonstrated by experimental results. In the article [9] the problem of autonomous collision-avoidance of mobile robots in dynamic environments is considered. The system includes a threat-avoidance unit based on the neural estimation of threat degree.

The articles [10-18] present the results based on using the risk function.

The possibilities of using generalized risk functions to solve optimization tasks in case of uncertainties are thoroughly observed in the general review [10].

The work [11] sets the optimizations problems of minimizing the integral of that function along the target trajectory, basing on the definition of the risk function, which is set in each point within the space where the sources are located. This trajectory can be imposed with various restrictions, among which the minimum trajectory length *l* and the presence of an upper limit of this length are particularly important.

The authors use the risk function for finding an object at the point *M* from the sources action *K* in the form of

$$r(M) = \sum_{k=1}^{K} \sigma_k \, / \, d_k^2(M),$$

where σ_k is the risk weighting of the k[th] source with the center at the point $O^{(k)}$; $d_k(M)$ is the distance between $O^{(k)}$ and M. Next, the indicated optimization problem is set with the constraint $l \le l_{\min}$, where l and $l_{\min}$ are the length of the optimized trajectory and its maximum permissible length. In the case of a single source, this problem can be solved analytically by the methods of variations calculus. For the case of many sources, the authors proposed the nonlinear programming algorithm based on the optimization of finite transitions between the nodes of a special graph that takes into consideration the weight coefficients of edges in the form of distances between the adjacent vertices and the respective transmission costs in the form of the risk function integrals on these edges.

The papers [12, 13] show that in case of an object, moving with changing speed in the neighborhood with one source, the optimal trajectory and speed-changing mode are such that the current value of the summarized "signal" from the source must be constant in time. In the papers [14 – 16] this property is generalized in the cases of several sources, moving and heterogeneous observers.

In [17], the authors introduce a functional that estimates the risk of detecting the j^{th} moving object of a group that operates jointly (for example, to break through the enemy defenses) by one of the N sensors that can summarize the signals incoming at the same time:

$$R_j = \int_0^{T_j} \sum_{i=1}^{N} \frac{\left(v_j(t)\right)^m}{\left(\rho_{ij}(t)\right)^k} dt,$$

where $\rho_{ij}(t)$ and $v_j(t)$ are the distance to the i^{th} sensor and the velocity of the j^{th} moving object of a group; T_j is the approximate time of this object to reach its target point. Here the "signal" on each sensor at a given moment of time reflects the respective summand, which depends from $\rho_{ij}(t)$ and $v_j(t)$. The exponent k characterizes the physical field in which the detection is carried out, and the exponent m is the dependence of the emitted signal's intensity level from the object's velocity. Thus, for example, for a primary hydroacoustic field in a shallow sea $k = 1$; $k = 2$ corresponds to a thermal field, a primary electromagnetic field or a primary hydroacoustic field in the deep sea; for the magnetic field detectors $k = 3$, and in case of the active detection mode for electromagnetic or hydroacoustic static fields $k = 4$.

Next, the various modifications of the introduced functionality are introduced to build the behavior models for the moving object groups, which differ by the information organization of their elements, i.e. by their degree of awareness about each other's actions and sensors. The resulting functionals already take into account the mentioned organization. There are the given results of a simulation of an enemy defense breakthrough by the groups with the different informational organization. The common aspect of all models constructed this way is the operation with the concept of detection risk instead of the concept of detection probability.

For example, introduced by the authors for the "non-cooperative" model risk function:

$$r(x,y) = \min\left\{ \sum_{i=1}^{N} \frac{c_i}{\left(\rho_i(x,y)\right)^k} ; 1 \right\},$$

where $\rho_j(x,y)$ is the distance from the current location of the object to the i^{th} sensor does not correctly reflect any probability. First, the necessity to exclude the first argument of the function min(, 1), the absence of smoothness of this dependence on of the object's position indicates the artificial nature of approximation of the true probability function of the considered event – the detection of a moving object by sensors at a given point. Second, the introduced function does not allow us to evaluate the probabilities of detection by sensors or the probability of damage from fire sources or other adverse factors of the enemy when the assumption of the instantaneous action of these sources of counteraction has already been incorrect. Thus, the model constructed by the authors does not permit to accumulate correctly the probability of damage/detection during the final mission time in the general case.

In [18], the authors enhance the problem set in [17] by introducing the factor of concentration of individual moving objects in a small area, which increases the overall risk of their detection by the sensor system. The p_i value, introduced here by the authors, formally reflects the probability of detecting a moving object by a group of sensors under conditions of finding other objects is also introduced by the cut-off principle of the subintegral expression of the respective risk function using the function min(, 1). The authors themselves notice that the issue of the correct representation of this probability has not been fully considered by them.

In addition, please, note that the problem of estimation of the probability of damaging the moving object in case of its detection is not raised in [17, 18] as well.

The common thing of works [10-18] is the use of the risk function and absence of methods to calculate the probabilities of spotting, damage, etc. For example, an extreme trajectory, which can be obtained by an effective numerical method for optimizing risks on a network indirect graph [10], needs to determine this probability. Indeed, knowing the optimal trajectory dos not yet guarantee the completion of the mission by a single mobile object or a group of mobile objects if there is no way to calculate the probability of completing a mission. Having a methodology for calculating the probability of completing a mission, which is connected with passing the trajectory, will allow us in some cases to choose acceptable trajectories.

The papers [17, 18] use the concept of the probability of object's damage in case of its detecting, but this probability is not considered as a function of trajectory and/or moving speed, because it is set as an external constant.

The papers [19-22] also present different methods of optimizing the risk function in the conflict environment. The method of planning of robot's behavior in an environment with threats is developed in the article [19]. It proposes the bioinspired system that realizes the global and local robot's behavior. On the local level, the risk function is used, which is optimized with the proposed AI system. Also, in the mentioned paper, the collision probabilities are not introduced, and the risk function depends on distances and experimentally adjusted parameters. In the article [20] the problem of visiting the target points by one or several robots in the hostile environment is solved. Heuristic algorithms for obtaining optimal paths from the point of view of the probability of the task are proposed in the article. The article uses probabilities set by default, which calculation method is not proposed.

The study [21] considers the problem of finding a reliable path in an uncertain conflict environment, which is solved by minimizing the expected risk. To solve the problem, a topological map is compiled.

The authors of [22] propose the method of path planning in the stochastic environment. This method maximizes the probability of arriving at a given point within the given time interval. The authors describe the probability of passing the area using Levi's distribution. The problem is solved for an environment represented by the graph.

Thus, the problem of calculation of passing probability for random trajectories in conditions of random sources, also including their heterogeneity, is relevant and not well studied.

It should be mentioned that the problem of moving in the area with the sources that create threats for completing the assigned mission has much in common with movement in the environment with the obstacles. To solve this problem a significant number of methods are used which are based on potential fields [23-26], dynamic (including unstable) forces [4, 5, 27], graph-searching algorithms [28-31], and assessment of geometrical complexity of the environment [32].

Potential fields allow us to find the optimal path with a rigorous justification of the stability of the impact of the introduced virtual forces. However, the method restrictions are that it can guarantee obstacle avoidance, but the particular look of the planned trajectories cannot be exactly forecasted.

Dynamic (including unstable) fields make it possible to efficiently solve the problem of local minima; however, predicting the robot's trajectory in advance also seems problematic.

In many cases the discretized graph-searching algorithms allow us to find paths in the conflict and uncertain environments more effectively. Thus, [28] presents the algorithms of calculating the shortest path basing on the piecewise-linear discrete model of the configuration space, which permits to use it for random topology and dimensionality. The proposed method builds the geodesic path in the metrical space more precisely than the Dijkstra's algorithm.

The article [29] considers the multicomponent procedure that includes ant colony optimization and A*. The 3-D space is considered. There is a given set of target points for which the ant colony optimization calculated the sequence of passage on large cells. The path plan is clarified using A*, which takes into account the local obstacles.

The paper [30] presents the algorithm of planning in the 3-D environment that is presented in the form of the probability graph. To build the graph, a sufficient number of points-nodes are generated, the coordinates of which are random variables with a uniform distribution law inside an admissible region of space. On this graph, the permissible movements are highlighted without taking into account the obstacles. Then, using the algorithm A* and potential fields, the obstacles are taken into account and the path is formed.

The discrete searching procedures allow us to find the suboptimal paths with insignificant calculation expenses, but the obtained trajectory may need further processing, for example, path smoothing. From this point of view, the continuous methods can at once take into account the characteristic peculiarities of passing the conflict areas by the robots.

The present article is organized in the following way. Section 2 formulates the problem to be solved and gives a probabilistic description of the source with a local field of action. The description of the characteristic probability function is introduced and the basic assumptions are made. In section 3, based on the assumptions made, the characteristic probability function in continuous form for one source is proposed. In section 4, based on the finite-difference introduction, the function is introduced, which allows us to approximately calculate the probability of robot's non-detecting when it is moving by a random trajectory in the area of the source. The obtained result is generalized in the case of several repeller sources. Also, the expression for describing the probability of a robot's non-detecting is given in continuous form. Section 5 presents the examples of optimization problems connected with robot's movement within the area of one or several repeller sources. Chapter 6 contains the examples of numerical simulation.

2. Problem statement. Let us assume that each repeller source $\tilde{S}$ has a center O in small neighborhood of which its radiation is maximal. For definiteness, we assume that the source creates probing radiation and we estimate the probability of the object passing undetected in the field of this source. However, all subsequent conclusions can be transferred to the sources of other types, in particular, creating damaging factors.

We will single out the sources whose scope is the entire subspace of the Euclidean plane that is valid for laying target trajectories of a continuous type, and sources with a scope limited by some figure inside the specified subspace, for example, by a circular sector.

Basing on the information about technical characteristics of this source it is possible to construct a dependence of the probability $q_s(Tr)$, $q_s \le 1$ of a mobile robot passing undetected/undamaged from the particular trajectory Tr defined as a vector function of time. We will consider only the trajectories that can be presented as a set of a finite number of parameters.

Let us divide the initial object's movement trajectory Tr by separate sections Δr_i, $i=1,2,\ldots,N$; each section corresponding to an elementary trajectory (Tr_i).

We introduce the following probabilistic space (Ω_{Tr}, $\mathbb{F}$, P), where Ω_{Tr} is the set of elementary events corresponding to the given trajectory Tr, $\mathbb{F}$ is the admissible set of the subsets Ω_{Tr}; P is the countably additive function on the set $\forall \mathrm{F} \in \mathbb{F}$: $\mathrm{P(F)} \le 1$, $\mathrm{P}(\Omega_{\mathrm{Tr}})=1$, $\mathrm{P}(\varnothing)=0$.

The set Ω_{Tr} contains of: A) finite set $\widehat{\Omega}_{Tr}$ of events $\omega(A_iA_{i+1}) \in \Omega$, $A_iA_{i+1} \equiv Tr_i \subset Tr$ – random elementary trajectory section (of the separation described above) of the type:

$$\omega(A_i A_{i+1}) = \left\{ \begin{matrix} \text{"}\textit{Object moving along the Tr will be } \text{detected} \\ \textit{on the section } A_i B_i \textit{ of the given trajectory Tr} \end{matrix} \right\}.$$

B) one elementary event ω^*:

$$\omega^* = \left\{ \text{"}\textit{Object moving along the Tr will not be } \text{det}\textit{ected} \right\}.$$

Thus, $\Omega_{Tr} = \widehat{\Omega}_{Tr} \cup \omega^*$.

Notice, that the probability value for each event of the type $\omega(A_iA_{i+1})$ is calculated according to the rule:

$$\mathrm{P}\left[\omega(Tr_i)\right] = q_S(AA_i) \cdot \left[1 - q_S(Tr_i)\right],$$

where $q_s(Tr_i)$ is the probability of the object's non-detecting on the elementary section A_iA_{i+1} of the trajectory Tr irrelatively to the already passed section AA_i of this trajectory; $q_s(AA_i)$ is the probability of the object's non-detecting when passing the section AA_i.

If for each section Tr_i we determine its passing probability $q_{si}(Tr_i)$ then the value of the probability function of passing the mobile robot *undetected* along the entire trajectory $q_s(Tr)$ is calculated using the expression [33]:

$$q_s\left(Tr\right)=\prod_{i=1}^{N} q_{si}. \tag{1}$$

It needs to be mentioned that for the sources of continuous type we should use the division mentioned above to the entire trajectory, since, according to the definition of such sources, the entire trajectory is within their scope (see Fig. 1). In case of discrete sources it is enough to divide by sections only those parts of the trajectory that are within the respective figure which is the combination of scopes of these sources (see Fig. 2).

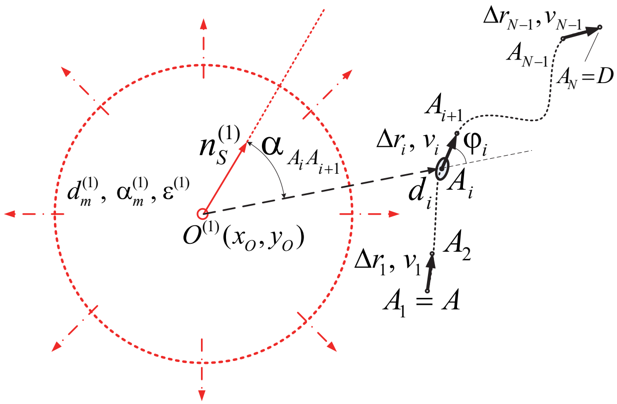


Fig. 1. Division of the final trajectory into elementary sections for the case of a continuous source: the entire trajectory is divided

In case of rather small sections Δr_i the probability $q_s(Tr)$ can be calculated using the limit

$$q_S(Tr)=\lim_{\left|\Delta r\right|_{\max}\to 0}\left[\prod_{i=1}^{N\left(\left|\Delta r\right|_{\max}\right)} q_{S,i}\right], \tag{2}$$

where $\left|\Delta r\right|_{\max}=\max\limits_{\{i\}}\left[\left|\Delta r_i\right|\right]$.

Thus, the problem can be reduced to determining the values of the function q_s on rather small sections of the trajectory Tr within each of which the change in the distance from the object to each source can be neglected, as well as the change of the object’s velocity.

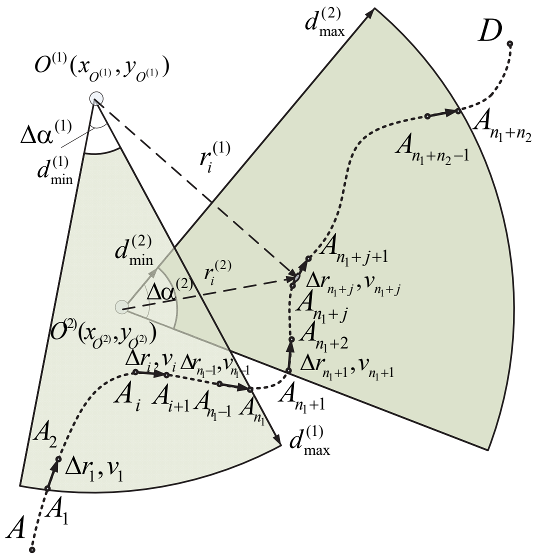


Fig. 2. Division of a finite trajectory into elementary sections for the case of discrete sources: in the general case, not the entire trajectory is divided

Assume that q_{si} is the monotonically increasing function of the distance d_i between the geometrical center M_i of the area Tr_i and the source's center O. This assumption is based on the fact that the probability of the robot spotting is monotonically increasing with decreasing distance to the center O. Assume that the source is directed. For simplicity, we assume that there is one characteristic direction of the maximum of its influence on the object and it is described by the unit vector n_s. If α_i is the angle between n_s and direction drawn from the source's center O to the point M_i, then the function q_{si} is monotonically increasing with the increase of the module α_i. In case of the discrete source we assume that the vector n_s simultaneously sets the direction of the axis of symmetry of the circular cone – the range of action of such source according to the accepted assumption.

Let us describe the characteristic change of the source's influence depending on the distance using the parameter d_m, and the respective change by angle using the parameter α_m.

Next, assume that the source has response inertia and detection error. The latter can be caused by the influence of interference and errors in the identification system associated with the source. Let ε be the effective frequency of the source, which is the greater, the lower the indicated inertia is. Thus, in case of fixed ε, the function q_{si} depends on the time T_i of passing the trajectory Tr_i. Let us assume that $q_{si}$is the monotonically decreasing function T_i. The bigger is, ε the lower is the probability of the objects' passing given the fixed time T_i of being within the source's field.

Notice, that in general case q_{si} depends on the orientation of the vector Δr_i in relation to the direction OM_i. However, in case of rather small Δr the influence of orientation can be neglected.

For the discrete type sources it might be appropriate to introduce the lower boundaries for values d and T: $d_{\min}$ и $T_{\min}$. For example, the restrictions on minimum detection distance can be connected with the peculiarities of the beam pattern. The minimum duration of the object's presence in the field of the source can be determined by the necessity of signal accumulation or inertia of identification or guidance systems.

Thus, we will characterize the sources of both types by the set of values: $\tilde{S}\{S,O,n_S,d_m,\alpha_m,\varepsilon\}$, moreover S (the scope) for the continuous type source coincides with the plane subspace indicated above, within which the movements of the object are permissible, and for the discrete type source it coincides with the indicated above circled cone with the center O, opening angle $\Delta\alpha^{(1)}$, directing vector of the symmetry axis n_s, minimum and maximum radiuses $d_{\min}$ and $d_{\max}$ ($d_{\min}$ can be zero).

Definition 1. The value $q_{0s}(d_i, \alpha_i, T_i)$ we will call the *characteristic probability function* of the source $\tilde{S}$, if for rather small sections Tr_i it determines the probability of their passing, i.e $q_{si}(Tr_i)=q_{0s}(d_i, \alpha_i, T_i)$.

3. Synthesis of the characteristic probability function. According to the Definition 1, the function $q_{0s}(d_i, \alpha_i, T_i)$ satisfies the following Statement 1.

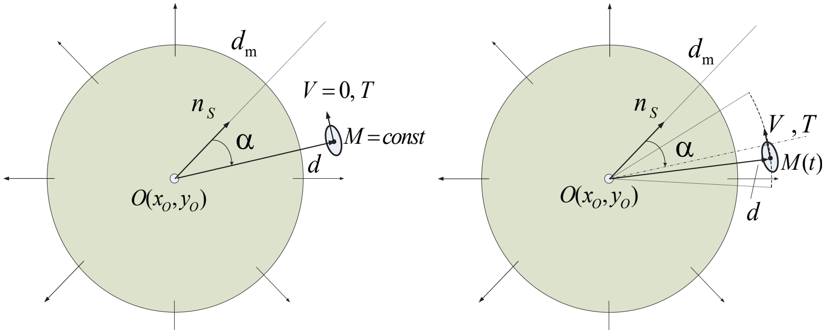
Statement 1. The probability of robot detection by the considered source remains unchanged in two cases:

1. The mobile robot is at rest for a time T at some point M determined by the distance d and the angle α;

2. The mobile robot is moving for a time T along the circular arc determined by the radius d and the angle α.

The indicated cases are presented on the example of continuous type source in Figure 3.

Let us introduce two events. Event 1 $Q_1(\tilde{S},d,T,\alpha)$ means that the robot is not detected while being at rest. Event 2 $Q_2(\tilde{S},d,T,\alpha)$ means that the robot is not detected while moving along the respective circular arc.



$|V| = 0$ – Object is not moved, $|V| = \alpha d / T$ – average velocity of the motion,

T – time of the finding at rest inwardly area S. T – time of the motion inwardly area S

Fig. 3. Conditions for the constancy of the probability of detecting a robot appearing in Statement 1

Radial arrows in Figure 3 show that the source's scope is exceeding the circle with the radius d_{m} that has the meaning of characteristic influence of the source over the distance to the object.

According to the definition 1, statement 1, as well as the assumptions made in section 2 regarding the properties of the function $q_{0\mathrm{s}}(d, \alpha, T)$ on rather small trajectory Tr, we propose the following function:

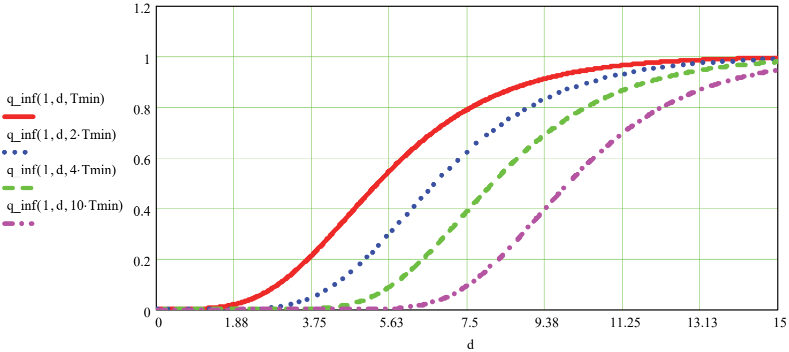
$$q_{0S}(d, T, \alpha) = \exp\left\{-\varepsilon T \exp\left[-(d / d_m)^{2p} - |\alpha / \alpha_m|^{2s}\right]\right\}, \tag{3}$$

where $p, s \in R_+$; values d_{m} and α_{m} are the normalizing divisors for the distance and the angle, respectively.

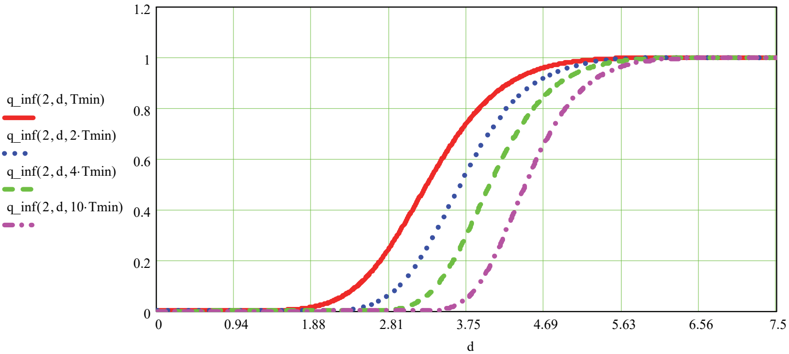
If we accept the hypothesis of a uniform distribution of the probability of non-detection of the robot in azimuth, then expression (3) will take the form:

$$q_{0S}(d, T) = \exp\left\{-\varepsilon T \cdot \exp\left[-(d / d_m)^{2p}\right]\right\}, \tag{4}$$

Figure 4 shows the dependencies of the characteristic probability function of the type (3) at α=0 from the distance d for different time spent by the robot in the field T=1, 2, 4, 10 sec, 2p=2, 2p=4, d_m=2 m, ε=10 sec $^{-1}$.



a) 2p=2



б) 2p=4

Fig. 4. Characteristic probability function

4. Probability of non-detection of the robot moving along random trajectory in the field of one or several sources. Let us consider the robot's movement along the random trajectory *AD* (Fig. 1), which travels through the scope of the source.

Let us divide the part of the trajectory into rather small intervals $A_iA_{i+1}=\Delta r_i$, $i=1,2,\ldots,N-1$. Obviously, for sufficiently big N we can neglect the change of the distance d and approximately assume:

$$q_{Si}(A_iA_{i+1}) \approx q_{0S}(d_i, |\Delta r_i / v_i|, \alpha_i), \tag{5}$$

where $q_{si}(A_iA_{i+1})$ is the desired probability of the object passing the area A_iA_{i+1} undetected; (d_i, Δr_i, v_i) are the distance to the source, vector of length, and velocity module of the i^{th} section of the trajectory.

For the correct use of the indicated approximation, we require that for each Δr_i the following condition is fulfilled:

$$\left|\Delta r_i\right|\cos\varphi_i \le k_d d_{\max}, \quad (6)$$

where k_d<<1 is the parameter that characterize the allowable maximum value of projection Δr_i on the на radius-vector connecting the cource's center with the initial point A_i of the area A_iA_{i+1}; φ_i is the angle between the direction from the source to the point A_i and the directed segment Δr_i.

The fulfillment of condition (6) allows us to use of the expression (5) for an approximate calculation of the probability of the robot non-detection.

If the obtained division fulfills the condition (6) then, according to the product theorem of event probabilities following one another [33], we obtain that the probability $q_S(AD)$ of the robot's passing along the trajectory AD, taking into account (5), (6) and (3), determined in the following way:

$$\begin{gathered} q_S(AD) = \prod_{i=1}^{N-1} q_S(A_iA_{i+1}) \approx \prod_{i=1}^{N-1} q_{0S}(d_i, \left|\Delta r\right|_i / v_i, \alpha_i) = \\ = \exp\left[-\varepsilon\cdot\sum_{i=1}^{N-1}\left(\left|\Delta r_i\right|/v_i\right)\cdot\exp\left[-\left(\frac{d_i}{d_m}\right)^{2p} - \left|\frac{\alpha_i}{\alpha_m}\right|^{2s}\right]\right]. \end{gathered} \quad (7)$$

The expression (7) can be generalized in case of a finite number of sources of the observed type under the assumption of their independence from each other. Suppose there are K sources $\tilde{S}^{(k)} = \tilde{S}^{(k)}\left\{S^{(k)}, O^{(k)}, n_S^{(k)}, d_{\mathrm{m}}^{(k)}, \alpha_{\mathrm{m}}^{(k)}, \varepsilon^{(k)}\right\}$, k=1,2,…,K. In this case, the probability of the robot's passing undetected along the trajectory AD is determined by the expression

$$\begin{gathered} q_S(AD) \approx \prod_{k=1}^{K} q_{S^{(k)}}(AD) = \\ = \exp\left[-\sum_{k=1}^{K}\varepsilon^{(k)}\cdot\sum_{i=1, i\in\left\{N^{(k)}\right\}}^{N-1}\left(\left|\Delta r_i\right|/v_i\right)\cdot\exp\left[-\left(\frac{d_i^{(k)}}{d_m^{(k)}}\right)^{2p} - \left|\frac{\alpha_i^{(k)}}{\alpha_m^{(k)}}\right|^{2s}\right]\right], \end{gathered} \quad (8)$$

where $q_{S(k)}(AD)$ is the probability of the robot's passing undetected by the k^{th} source. It is assumed here that to calculate each probability $q_{S(k)}(AD)$ the initial trajectory is divided by the same system of points $A_iA_{i+1}=\Delta r_i$, $i=1,2,\ldots,N-1$ that fulfill the correlation (5) for each source (see Fig. 3); in case of fixed k the summation over i is carried out over the subset of indices $i \in \{N^{(k)}\}$, which correspond only to the sections of the division within the area $S^{(k)}$. All other values with the upper index k have the meaning similar to the considered above for one source.

Now let us obtain the expression for the probability of non-detection of the robot moving along the random trajectory in the continuous form. By tending the upper limit for the lengths of division sections in (7) to zero, in the limit we transform (7) to the following expression:

$$q_S(Tr) = \\ = \exp\left[-\varepsilon \cdot \lim_{|\Delta r|_{\max}\to 0} \sum_{i=1}^{N(|\Delta r|_{\max})-1} \Delta t_i \cdot \exp\left[-\left(\frac{d_i}{d_m}\right)^{2r} - \left|\frac{\alpha_i}{\alpha_m}\right|^{2s}\right]\right], \quad (9)$$

Where $\Delta t_i = |\Delta r_i| / v_i$ is the time of object's movement on the elementary linear sector of the trajectory Δr_i with velocity v_i.

From the expression (9) we obtain:

$$q_S(Tr) = \exp\left[-\varepsilon \cdot \int_{t_A}^{t_D} \exp\left[-(d(t)/d_m)^{2r} - |\alpha(t)/\alpha_m|^{2s}\right] dt\right], \quad (10)$$

where $d(t) = \sqrt{[x(t)-x_O]^2 + [y(t)-y_O]^2}$, $\alpha(t)=acos(ns,\ OM(t)/\|OM(t)\|)$, $M(t)=[x(t);\ y(t)]$, $O=[x_O; y_O]$; t_A, t_D are the moment of the robot being at the starting (A) and finishing (B) points.

For the source with the function $q_{0s}(d, \alpha, T)$ that does not depend on the azimuth, basing on the expression (4), in the same way we obtain

$$q_S(Tr) = \exp\left[-\varepsilon \cdot \int_{t_A}^{t_B} \exp\left[-(d(t)/d_m)^{2r}\right] dt\right]. \quad (11)$$

5. Optimization problems of the robot passing in the field of one or several sources. 5.1. The problem of movement in the field of the sources that does not depend on the angle α. Let's consider the case of several continuous type sources, where each source is described by the characteristic probability function of the type (4).

Let us assume that the movement is made along the piecewise-linear trajectory *Tr* with the points $\{A_l\}$, l=1,2,…,N_0 and constant velocity *v*.

For several independent sources the probability of passing the considered trajectory without being detected $q(Tr)$ is determined by the following multiplication

$$q(Tr) \equiv \prod_{k=1}^{K} q_S^{(k)}(Tr), \tag{12}$$

where $q_S^{(k)}(Tr)$ is the probability of passing the trajectory under influence of the source $\tilde{S}^{(k)}$.

Considering the expressions (11) and (12) and the fact that at a constant movement velocity it is possible to go from the differential of time to the differential of movement, for the piecewise-linear trajectory we obtain:

$$q(Tr) = \exp\left[-\sum_{k=1}^{K} \varepsilon^{(k)} \cdot I^{(k)}\right], \tag{13}$$

$$I^{(k)} \equiv \frac{1}{v} \sum_{l=1}^{N_0-1} \int_{0}^{|A_l A_{l+1}|} \exp\left[-\left(d_{(l)}^{(k)}(\rho) / d_m^{(k)}\right)^2\right] d\rho, \tag{14}$$

where $\varepsilon^{(k)}$ and $d_m^{(k)}$ are the characteristic frequency and distance of the k^{th} source; $d_{(l)}^{(k)}(\rho)$ is the distance from the center of the k^{th} source to the integration point on the linear section $A_l A_{l+1}$. The value of $d_{(l)}^{(k)}(\rho)$ is calculated in the following way:

$$d_{(l)}^{(k)}(\rho) = \sqrt{\rho^2 + \left(d_l^{(k)}\right)^2 + 2 d_l^{(k)} \rho cos\varphi_l^{(k)}}, \tag{15}$$

where $d_l^{(k)} = \left|O^{(k)} A_l\right|$; $\varphi_l^{(k)}$ is the angle between the direction $O^{(k)}A_l$ and the vector $A_l A_{l+1}$. The respective geometrical constructions are given in the Figure 5.

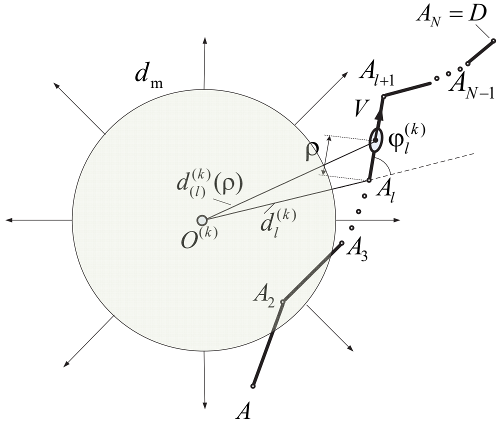


Fig. 5. Determination of the characteristic geometrical values

By performing integration in (14), the summation over l can be converted to:

$$I^{(k)} \equiv \frac{\sqrt{\pi} d_m^{(k)}}{2v} \sum_{l=1}^{N_0-1} \left\{ \exp\left[-\left(\frac{d_l^{(k)} \sin\varphi_l^{(k)}}{d_m^{(k)}} \right)^2 \right] \cdot \right. \tag{16}$$

$$\left. \left[sign\left(\tilde{\rho}_{2,l}^{(k)}\right) erf\left(\left|\tilde{\rho}_{2,l}^{(k)}\right|\right) - sign\left(\tilde{\rho}_{1,l}^{(k)}\right) erf\left(\left|\tilde{\rho}_{1,l}^{(k)}\right|\right) \right] \right\},$$

$$\tilde{\rho}_{1,l}^{(k)} = \frac{d_l^{(k)} \cos\varphi_l^{(k)}}{d_m^{(k)}},\ \tilde{\rho}_{2,l}^{(k)} = \tilde{\rho}_{1,l}^{(k)} + \frac{\rho}{d_m^{(k)}}. \tag{17}$$

Here we use the designation of the probability integral [34]:

$$erf(\mathrm{x}) = \left(2/\sqrt{\pi}\right) \int_0^x \exp(-\xi^2)\, \mathrm{d}\,\xi. \tag{18}$$

It is obvious that the function (13) on the set of all possible piecewise-linear trajectories has no global extremum if the number of sources of the observed type is finite. Therefore, the quality functional should include, along with (13), at least one more component. Let such component describe the the requirement of minimizing deviations from the given initial trajectory $Tr^{(0)}$ with the fixed endpoints $A_1=A$, $A_{N0}=D$. Then we can propose the following optimization function:

$$G(Tr)=\delta d_{m0}^{2}q_{S}(Tr)-(1-\delta)\rho^{2}(Tr,Tr^{\langle 0\rangle})\to \max,\delta\in(0,1), \tag{19}$$

where $d_{m0}=\sum_{k=1}^{K}d_{\max}^{(k)}/K$ is the normalizing factor for equalizing the dimensions and orders of magnitude of the terms in (19); the value ρ is the distance between the target and the initial trajectory described by the coordinates A_{l0}:

$$\rho(Tr,Tr^{(0)})\triangleq\sqrt{\sum_{l=1}^{N_0}\left(x_l-x_{l0}\right)^2+\left(y_l-y_{l0}\right)^2}, \tag{20}$$

$A_{l0}=\left(x_{l0},y_{l0}\right), l=2,..,N_0-1$ are the coordinates of the points of the initial trajectory of the null approximation; $A_l=\left(x_l,y_l\right)$ are the coordinates of the points of the trajectory. Parameter $\delta\in(0,1)$ characterizes the weights of each component of the criterion (19).

Let us consider an approximate solution to the problem of optimal motion in the field of repeller sources according to the criterion (19).

Due to the high complexity of the function $q_s(Tr)$ in the functional (19), we will consider the other function, which behavior is close to the $q_s(Tr)$. As such a function, we choose the distance from the current point to the geometric center of the repeller sources $\tilde{O}$.

Then the functional (19), taking into account the expression (20) takes the form:

$$G^{*}\left[\left\{X^{l}\right\}\right]=\delta\sum_{k=1}^{K}\sum_{l=2}^{N-1}\left|A_l-O^{(k)}\right|^2-\left(1-\delta\right)\sum_{l=2}^{N-1}\left|A_l-A_{l0}\right|^2. \tag{21}$$

After calculating the partial derivatives of the function (21), using the coordinates (x_l, y_l), we find the stationery point A_l:

$$A_{l1}=\frac{\delta K\tilde{O}-\left(1-\delta\right)A_{l0}}{\delta\left(K+1\right)-1},$$
$$\tilde{O}=\sum_{k=1}^{K}O^{(\mathrm{k})}/K. \tag{22}$$

Index 1 in the expression (22) means the first iteration of finding the stationery point.

After calculating the second derivative of the expression (21), let us find the condition for local maximum in the form of

$$\delta < 1/\left(K+1\right). \qquad (23)$$

In case of fulfillment of the condition (23), according to the Sylvester's criterion [35], we have the alternating signs of all the subminors of the corresponding Hessian matrix, starting with the negative one, which is easily established due to the diagonal form of this matrix. This indicates that at the point (22) there is the local maximum of the function (21). Moreover, this maximum is global, because this function has no other stationery points besides (22). That is why $G^{*}\left[A_{l1}\right] > G^{*}\left[A_{l0}\right]$. Points A_{l1} are the coordinates of the suboptimal trajectory of the first iteration.

If we now consider the trajectory of the first iteration as the original trajectory, and apply the transformations described above, then we obtain the expression:

$$A_{l2} = \frac{\delta\left[\delta\left(K+1\right)-1+\delta-1\right]K\tilde{O}-\left(1-\delta\right)^{2}A_{l0}}{\left[\delta\left(K+1\right)-1\right]^{2}}. \qquad (24)$$

Points A_{l2} are the coordinates of the suboptimal trajectory of the second iteration.

By carrying out further calculations using the method of mathematical induction we obtain the following expression:

$$A_{lu} = \frac{\delta\sum\limits_{m=1}^{u}\left\{\left[\delta\left(K+1\right)-1\right]^{u-m}\left(\delta-1\right)^{m-1}\right\}K\tilde{O}+\left(\delta-1\right)^{u}A_{l0}}{\left[\delta\left(K+1\right)-1\right]^{u}}. \qquad (25)$$

By simplifying the last expression, we obtain:

$$A_{lu} = \tilde{O}+\left(A_{l0}-\tilde{O}\right)s^{u}, \qquad (26)$$

$$s = \frac{1-\delta}{1-\delta(\mathrm{K}+1)}. \qquad (27)$$

Under the condition (24) the expression (27) is greater than one, therefore, with an increase in the iteration number, the coordinates A_{lu} becomes more distant from the geometric center of the sources $\tilde{O}$.

We show that as the iteration number u increases, the first term of the criterion (21) monotonically increases. Let us consider the first term of the criterion (21) as the function of parameter b=s^{u}. By substituting the expression (26) to the first term (21) we obtain:

$$G_1^*(b) = \delta\sum_{k=1}^{K}\sum_{l=2}^{N-1}\left|\tilde{O} + \left(A_{l0} - \tilde{O}\right)b - O^{(k)}\right|^2. \tag{28}$$

By differentiating (28) we obtain:

$$\partial_b\left[G_1^*(\mathrm{b})\right] = 2b\sum_{k=1}^{K}\sum_{l=2}^{N-1}\left|A_{l0} - \tilde{O}\right|^2. \tag{29}$$

According to (29), the function (28) reaches the minimum at the point b=0 and monotonically increases as b increases. Moreover, as the iteration number increases, the probability $q_s(Tr)$ in (19) should also increase, since the increase of $G_1^*(\mathrm{b})$ means distancing the path points from the sources. In this regard, we consider the asymptotic behavior of the function $q_s(Tr)$.

Note, that the coordinates of all points A_{lu}, i=1,2,N-1, in accordance with (26) and when u→∞ are infinitely increasing, so all the segments of the trajectory A_{lu}, except the first and the last one, are asymptotically leaving the scope of repeller sources. This corresponds to the fact that in the sum (16) for each source it is necessary to take only the first and last terms.

We consider these terms at the iteration u, and then move on to the corresponding limits for u→∞. For the first term we obtain:

$$\lim_{u\to\infty}\left[\cos\varphi_{1,u}^{(k)}\right] \triangleq \cos\varphi_{1,\lim}^{(k)} = \lim_{u\to\infty}\left[\frac{\left(O^{(\mathrm{k})}A, \tilde{O} + \left(A_{20} - \tilde{O}\right)b\right)}{d_1^{(\mathrm{k})}\left|\tilde{O} + \left(A_{20} - \tilde{O}\right)b\right|}\right] =$$

$$= \frac{\left(O^{(\mathrm{k})}A, A_{20} - \tilde{O}\right)}{d_1^{(\mathrm{k})}\left|A_{20} - \tilde{O}\right|},\ \cos\varphi_{1,u}^{(k)} = \frac{\left(O^{(\mathrm{k})}A, AA_{2u}\right)}{\left|O^{(\mathrm{k})}A\right|\left|AA_{2u}\right|},$$

$$\tilde{\rho}_{1,l,u}^{(k)} = \frac{d_l^{(k)}\cos\varphi_l^{(k)}}{d_m^{(k)}},\ \lim_{u\to\infty}\left[\tilde{\rho}_{1,l,u}^{(k)}\right] = \frac{d_1^{(k)}\cos\varphi_{1,\lim}^{(k)}}{d_m^{(k)}};$$

$$\rho_{1,u} \to \infty,\ \tilde{\rho}_{2,1,\mathrm{u}}^{(k)} = \tilde{\rho}_{1,1,\mathrm{u}}^{(k)} + \frac{\rho_{1,u}}{d_m^{(k)}} \to \infty.$$

Taking into account the latter expressions and the properties of the function (18) $erf(\infty)=1$, we obtain:

$$I_I^{(k)} = \frac{1}{v}\exp\left[-\left(\frac{d_1^{(k)}\sin\varphi_{1,\lim}^{(k)}}{d_m^{(k)}}\right)^2\right]\cdot$$
$$\cdot\left[1-sign\left(\cos\varphi_{1,\lim}^{(k)}\right)erf\left(\left|\frac{d_1^{(k)}\cos\varphi_{1,\lim}^{(k)}}{d_m^{(k)}}\right|\right)\right]. \tag{30}$$

Similarly, for the last term of the sum (16) we calculate:

$$\cos\alpha_{\mathrm{N}-1,u}^{(k)} = \frac{\left(O^{(\mathrm{k})}D, O^{(\mathrm{k})}A_{N-1u}\right)}{\left|O^{(\mathrm{k})}D\right|\left|O^{(\mathrm{k})}A_{N-1u}\right|},$$

$$\lim_{u\to\infty}\left[\cos\alpha_{\mathrm{N}-1,u}^{(k)}\right] \triangleq \cos\alpha_{\mathrm{N}-1,\lim}^{(k)} = \frac{\left(O^{(\mathrm{k})}D, A_{N-10}-\tilde{O}\right)}{d_N^{(\mathrm{k})}\left|A_{N-10}-\tilde{O}\right|},$$

$$d_{N-1,u}^{(k)}\sin\varphi_{N-1,u}^{(k)} = d_N^{(k)}\sin\alpha_{\mathrm{N}-1,u}^{(k)} \to d_N^{(k)}\sin\alpha_{\mathrm{N}-1,\lim}^{(k)},$$

$$d_{N-1,u}^{(k)}\cos\varphi_{N-1,u}^{(k)} + \rho_{N-1,u} = -d_N^{(k)}\cos\alpha_{\mathrm{N}-1,\mathrm{u}}^{(k)} \to -d_N^{(k)}\cos\alpha_{\mathrm{N}-1,\lim}^{(k)}$$

$$\cos\varphi_{\mathrm{N}-1,u}^{(k)} = \frac{\left(O^{(\mathrm{k})}A_{N-10}, A_{N-10}D\right)}{\left|O^{(\mathrm{k})}A_{N-1u}\right|\left|A_{N-1u}D\right|} \to \frac{-\left(A_{N-10}, A_{N-10}\right)}{\left|A_{N-10}\right|^2} =$$
$$= -1 \triangleq \cos\varphi_{\mathrm{N}-1,\lim}^{(k)}.$$

$$I_{II}^{(k)} = \frac{1}{v}\exp\left[-\left(\frac{d_N^{(k)}\sin\alpha_{\mathrm{N}-1,\lim}^{(k)}}{d_m^{(k)}}\right)^2\right]\cdot$$
$$\cdot\left\{sign\left(-\cos\alpha_{\mathrm{N}-1,\lim}^{(k)}\right)erf\left(\left|\frac{d_N^{(k)}\cos\alpha_{\mathrm{N}-1,\lim}^{(k)}}{d_m^{(k)}}\right|\right)+1\right\}. \tag{31}$$

Then the limit value of the probability $q_{\mathrm{s},\lim}$ is determined by the expression:

$$q_{S,\lim} \triangleq \prod_{k=1}^{K} q_{S,\lim}^{(k)} = \exp\left[-\sum_{k=1}^{K}\varepsilon^{(k)}(I_I^{(k)} + I_{II}^{(k)})\right]. \tag{32}$$

Figure 6 shows the geometric constructions, basing on which the expressions and the passages to the limits were obtained in (30) and (31).

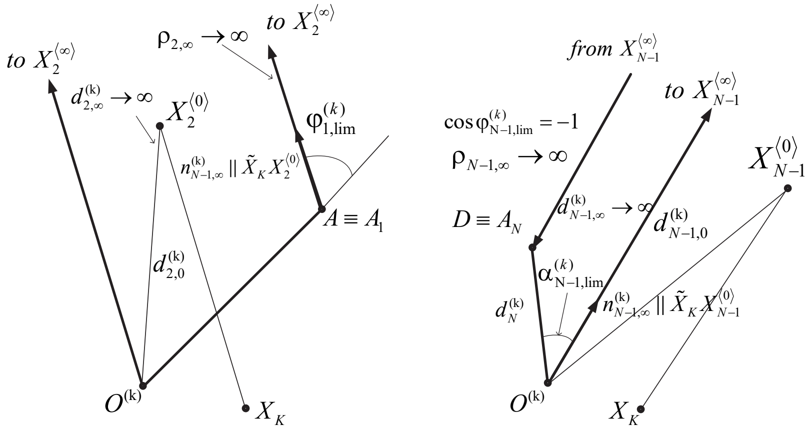


Fig. 6. Geometric constructions to the determination of values (30, 31)

Thus, the procedure (26) allows us to build up from the initial trajectory A_{l0} optimally by the criterion (21). That allows solving the problem of the path finding that insures the given probability of passing undetected by the sources and has minimum difference from the initially planned path. In this case, the robot trajectory belongs to the piecewise-linear linear class, and the movement velocity is assumed to be constant.

5.2. Problem of accounting for repeller sources on trafficability maps. The characteristic probability function for sources enables to account the repeller sources on trafficability maps, for example, during the path planning that uses procedures that divide space into cells [36-39]. To perform this it is necessary to endow each i^{th} cell of the map with a certain weight c_{i} that reflects the probability of passing this cell by an object that moves in the source's field. The invariance of the weights c_{i} relative to the movement trajectory is a necessary condition.

Let us assume that in some area $\text{U}\subset\text{R}_2$ there are K repeller sources and the obstacles. Depending on the locations of the obstacles, for each area of cell U there are given weights $c_i^{(1)}, i = 1,2,..,N$ that characterize the geometric passability, and N is the number of all cells. We assume that any robot's trajectory Tr completely belongs to the area U.

Let us consider the weight $c_i^{(2)}, i = 1,2,..,N$ that reflects the probability of the robot's detection in this cell. Then we can consider the total weight in the form of:

$$c_i = \delta c_i^{(1)} + c_{\max}^{(1)}(1-\delta)c_i^{(2)}, i = 1, 2, .., N, \qquad (33)$$

where $c_{\max}^{(1)}$ is the maximum allowable value of partial weight $c_i^{(1)}$; δ is the weight fraction of the first type in the total weight c_i.

In order to make weights $c_i^{(2)}$ invariant to the trajectory, let us make the following assumption. We assume that the robot passes through each cell during the same time T_e. This assumption is valid at a constant velocity v_e and the same cell sizes L_{in}. Then T_e is determined by the expression:

$$T_e = L_{in} / v_e. \qquad (34)$$

Assuming the characteristic size of the cell L_{in} sufficiently small compared to the distance from the sources to the path, we can replace the distance from the source to the path point by the distance from the source to the center of the cell, i.e.:

$$d_i^{(k)} = \left| O^{(k)} O_i \right|, \qquad (35)$$

where O_i is the center of the i^{th} cell.

Under the made assumptions, the probability of passing the i^{th} cell without detection takes the form:

$$q_i \simeq \prod_{k=1}^{K} \exp\left\{ -\varepsilon^{(k)} T_e \exp\left[-(d_i^{(k)} / d_m^{(k)})^{2r} - \left| \alpha_i^{(k)} / \alpha_m^{(k)} \right|^{2s} \right] \right\}. \qquad (36)$$

Then we choose the weight $c_i^{(2)}$ of each cell so that it decreases with increasing probability (36), i.e.:

$$c_i^{(2)} = 1 - q_i = 1 - e^{-\left(\sqrt{2}a/2v_e\right) \sum\limits_{k=1}^{K} \varepsilon^{(k)} \exp\left[-(d_i^{(k)}/d_m^{(k)})^{2r} - \left| \alpha_i^{(k)}/\alpha_m^{(k)} \right|^{2s} \right]}. \qquad (37)$$

Since the optimization algorithms build the trajectory that passes through the cells with minimum total weight, then the proposed method of forming weights takes into account both the geometric passability of the cells and the probability of detecting a robot in this cell.

Figure 7 gives a geometric explanation of the search for the optimal path using search procedures, for example, D*.

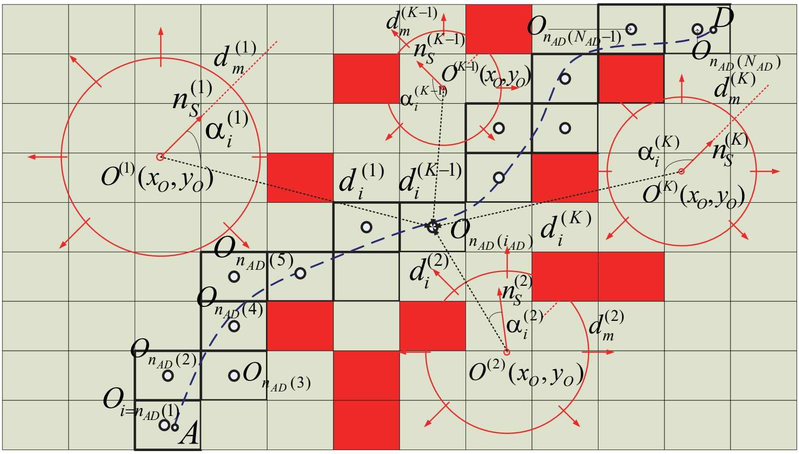


Fig. 7. Geometric explanations of the search for the optimal path in the environment with obstacles and repeller sources

Figure 7 shows the cells passed by the robot. For these cell the internal numeration $i=n(i_{AD})$ is introduced. It also shows four sources and their characteristics $d_i^{(k)}, \alpha_i^{(k)}$ that determine the influence on the robot in this cell. Red squares indicate the obstacles impassable for the object.

Thus, the correct estimation of the "average" probability of passing each partition cell by a moving object under the conditions of counteraction sources when solving the problems of path planning by the methods of the D* family of algorithms makes it possible to take into account the influence of these sources in the respective patency maps. Therewith, it is required to bring correctly to the same weight the two partial weights of each cell using a reasonable choice of parameters δ, $c_{\max}^{(1)}$ in the expression (33).

6. Example of identifying the parameters of the characteristic probability function. Let us consider an example of determining the parameters of the characteristic probability function (CPF) for the Abrams tank, which has the following specifications: the M256 directing gun; the rate of fire is η = 8 rounds per minute, effective firing range: 2500-3000 m. The average effective firing range is D_0=2750 m.

Let the "effective damage" mean damage with a probability from the range of $0.9 \le p \le 0.95$. Then the respective probability range for non-defeat is $q_2=0.05 \le q \le q_1=0.1$. The average probability of non-defeat is:

$$\langle q \rangle = (q_1 + q_2)/2 = 0{,}075. \tag{38}$$

In this case, the effective frequency of the source is its rate of fire: ε=η=8 rounds per minute.

We need to find the CPF in the form of (4) that does not depend on the azimuth angle.

First, let us consider the case when we take into account the requirement of the CPF at the distance D_0 to the object moving over time T=T_0=1 minute giving the value of the average probability of damaging from (38):

$$q_{0S}(D_0, T = T_0 = 1) = \langle q \rangle = 0,075. \tag{39}$$

Then the characteristic parameter of the effective range of the source d_m in terms of CPF, basing on (4) and (39), will be equal to:

$$d_m = D_0 / \sqrt{\ln\left[1/\ln\left(q_0^{-1}\right)\right]} = 2590 \;\; m. \tag{40}$$

The dependence $q_{0S}(d,T_0)$ is represented by the solid line in Figure 8.

Now let us consider the case when, in addition to the condition (39), it is necessary to ensure the given probability value in case of some distance D_{02}< D_0. For example, let D_{02} be the lower limit for the effective damage, and it is required to fulfill the condition:

$$q_{0S}(D_{02}, T_0) = q_2 \equiv 0,05. \tag{41}$$

Then, for the generality and accuracy of the CPF's approximation, we will consider the exponent $2p$ in the equation (4) as a real positive number.

If the frequency ε is fixed, then the conditions (39) and (41) generate two equations with respect to the parameters d_m and $2p$:

$$\begin{cases} \langle q \rangle = \exp\left\{-\varepsilon T \cdot \exp\left[-(D_0 / d_m)^{2p}\right]\right\}, \\ q_2 = \exp\left\{-\varepsilon T \cdot \exp\left[-(D_{02} / d_m)^{2p}\right]\right\}. \end{cases} \tag{42}$$

By introducing the new variables:

$$\mathrm{x} = \exp\left[-(D_0 / d_m)^{2p}\right],\; y = \left(D_{02} / D_0\right)^{2p}, \tag{43}$$

it is possible to rewrite the system (42) in the form of

$$\begin{cases} \langle q \rangle = \exp\left\{-\varepsilon T \cdot x\right\}, \\ q_2 = \exp\left\{-\varepsilon T \cdot x^y\right\}. \end{cases} \tag{44}$$

By solving the obtained system with respect to *x,y* and then returning to the variables d_{m} and $2p$, we obtain:

$$
\begin{gathered}
d_m = D_0 / \left[\ln\left(A^{-1}\right) \right]^{1/2p}, \;\; p = 0{,}5 \log_{\beta}\left(\log_A B\right), \\
A = \ln\left(\langle q \rangle^{-1}\right) / \varepsilon T, \;\; B = \ln\left(q_2^{-1}\right) / \varepsilon T.
\end{gathered}
\tag{45}
$$

When we make calculations (4) using the equations for the distance d_{m} and exponent $2p$ according to (45) and taking into account T=T_0=1min., ε= 8 rounds per minute, we obtain d_m=2531 m, 2p=1.449. The dependence of CPF is shown in Fig. 8 with the dash-dotted line.

If we reduce the probability value q_2, for example, to the level of q_2=0,03, then the corresponding calculation by formulas (4) and (45) will result in: d_{m}=2651 m, $2p$=3,282. The respective CPF dependence is shown in Figure 8 with the dashed line.

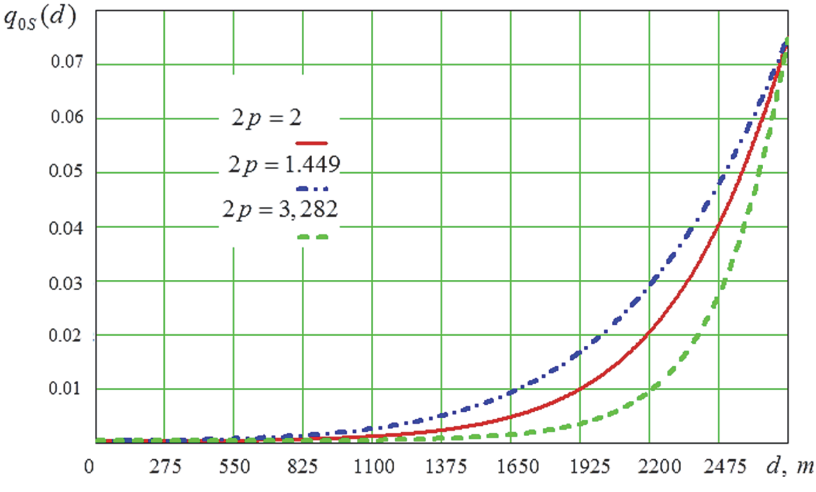


Fig. 8. Approximation dependences of the characteristic probability functions for the Abrams tank

The obtained results show the possibility to approximate the effective adjustment of the parameters of characteristic probabilistic functions to the already known or obtained data on the specifications that determine the probabilistic properties of this weapon. A similar technique for identifying the CPF parameters is also valid for other types of sources.

7. Numerical simulation results. During the simulation, the iterative procedure (26) has been investigated. The goal is to construct a trajectory that provides a given probability q_g and which deviates least from the original path. The problem of obtaining the highest possible probability of passing under fixed boundary conditions has also been studied.

In the simulation the criterion for robot stopping is the fulfillment of one of three conditions: 1) the passing probability q_g is reached; 2) the difference in the passing probabilities between the adjacent iterations is less than a given value ε_{kr}; 3) the maximum deviation of the average distance between the original and current trajectories ρ_{Trmax} is reached.

The total number of points of the piecewise-linear trajectory for all numerical experiments is N=5.

Figure 9 and Figure 10 present the simulation results for the following initial data (experiment “A”). Coordinates of the sources' centers: $O^{(1)}$=[2; 8]m, $O^{(2)}$=[4; 4]m, $O^{(3)}$=[8; 9]m, $O^{(4)}$=[10; 5]m. Effective radiuses of the sources: d_m= [2.5; 3.75; 3.75; 2.5]m. Characteristic frequencies of the sources ε=[10; 10; 10; 10] 1/sec. Stopping parameters at $k_\delta=\delta(K+1)=0.05$: q_g=0.9, ε_{kr}=0, ρ_{Trmax}=50m. Average velocity of passing the path v=10 m/sec.

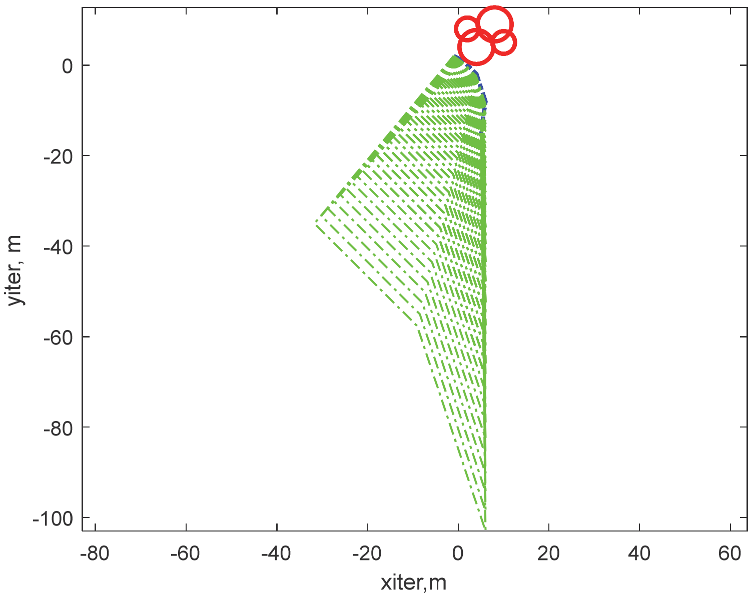


Fig. 9. Movement trajectories for the simulation of the experiment “A”

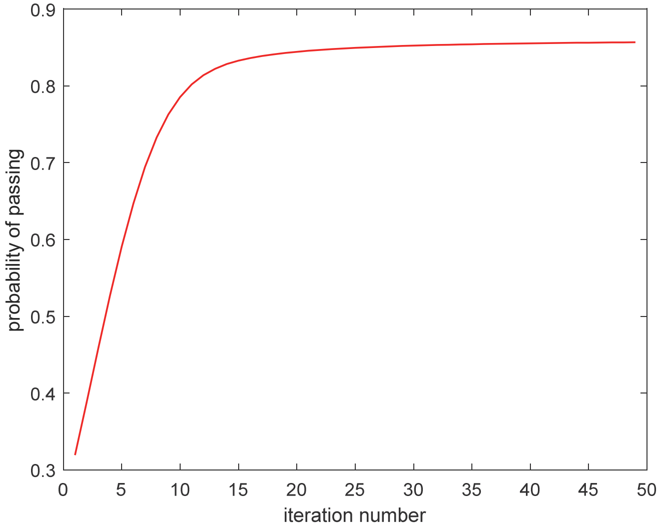


Fig. 10. The probability of passing in the simulation of the experiment "A"

The initial trajectory is marked in blue; the trajectories of the following iterations are marked in green. The probability of passing the initial iteration is q_0=0.2554. The probability of passing the trajectory in the last iteration is q=0.8567< q_g; The maximum possible probability is $q_{\lim}$=0.8588. Since $q_{\lim}$< q_g, the given probability of passing cannot be reached.

Now let us investigate the influence of the characteristic frequencies of the sources $\varepsilon^{(k)}$. To do this let's carry out the experiment "B", the conditions of which differ from experiment "A" by the values of the vector ε=[5; 5; 5; 5]1/sec. The probability of passing each elementary section of the trajectory should be higher than in case of the sources in the experiment "A". The simulation results are shown in Figure 11 and Figure 12.

In case of the experiment "B" the limit value of probability is increased in comparison with the experiment "A" $q_{\lim}$=0.9267. The given probability of passing the path undetected was reached in 12 iterations q=0.902>q_g. The probability of passing the initial trajectory is q_0= 0.505.

Now let us consider the case when the initial trajectory crosses the effective circle of at least one source. The conditions for the experiment "C" are the following. Coordinates of the sources' centers: $O^{(1)}$=[-1; 4]m, $O^{(2)}$=[4; 4]m, $O^{(3)}$=[8; -2]m, $O^{(4)}$=[10; 5]m, $O^{(5)}$=[-2; -1]m, $O^{(6)}$=[2; -6]m. Effective radiuses of the sources: d_m= [2.5; 3.75; 6.25; 2.5; 5; 3.75]m. Characteristic frequencies of the sources ε=[5; 5; 5; 5; 5; 5]1/sec. Stopping parameters at k_δ=0.15: q_g=0.9, ε_{kr}=0.0001, ρ_{Trmax}=50m. Average velocity of passing the trajectory v=10 m/sec.

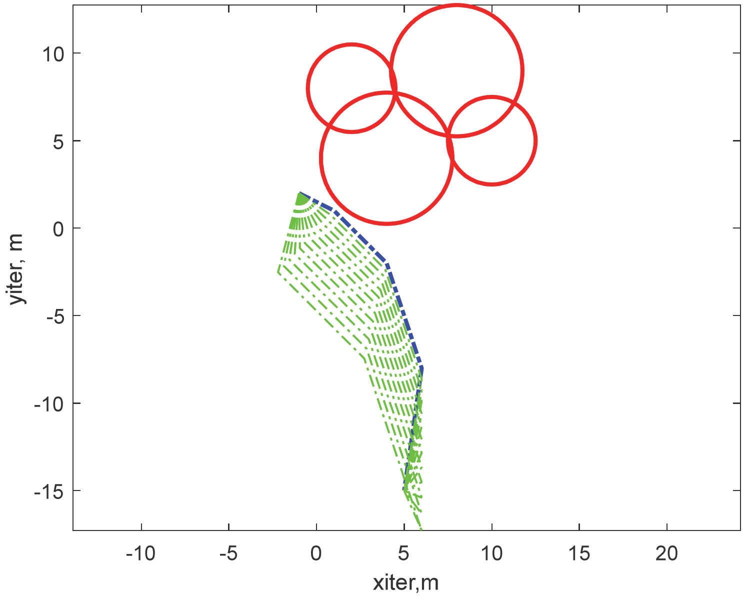


Fig. 11. Movement trajectories for the simulation of the experiment “B”

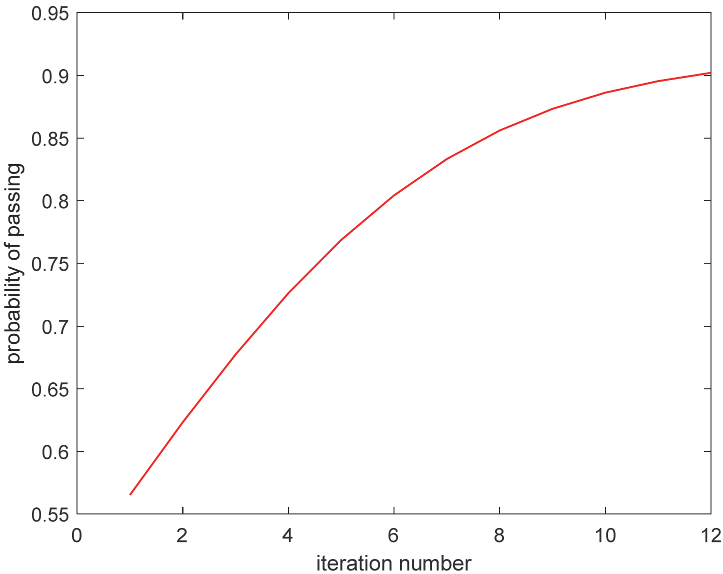


Fig. 12. The probability of passing in the simulation of the experiment “B”

The results are shown in Figure 13 and Figure 14. The probability of passing q=0.9>q_g is reached in 14 iterations. The probability of passing the initial path is q_0=1.6·10^{-4}. The maximum possible probability is q_{lim}= 0.987.

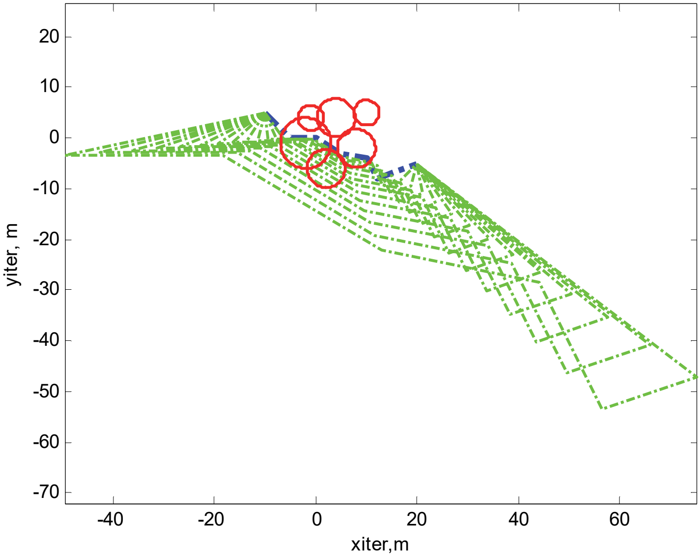


Рис. 13. Movement trajectories for the simulation of the experiment “C”

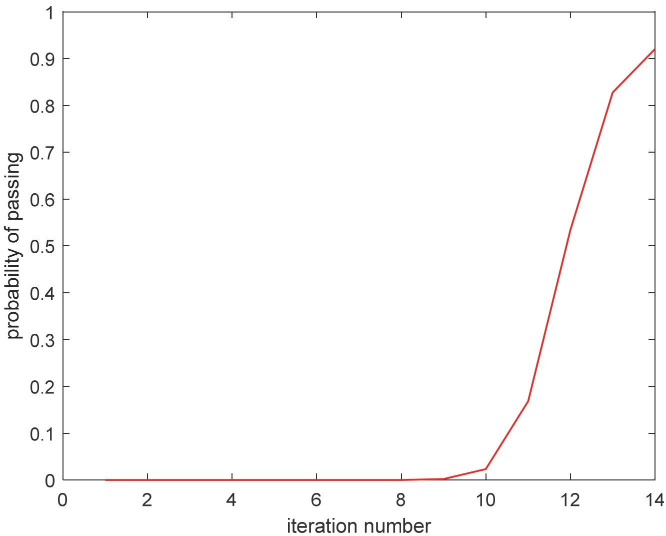


Fig. 14. The probability of passing in the simulation of the experiment “C”

In case of increasing the iteration number the monotonic increase of the passing probability is observed in all simulation results. This confirms the previously expressed qualitative consideration on the correlation of functionals (13) and (21). Moreover, a monotonic increase in these examples is observed near repeller sources, i.e. monotony is not only in the limit with sufficiently large numbers of iterations but also for small b.

However, in the general case, the behavior of the probability function of the passing from the iteration number near the sources may not be monotonic.

The comparison of the simulation results for cases “A” – “C” implies a significant influence of the initial trajectory on the behavior of the probability function $q_s(b)$. In particular, in Figure 13 and Figure 14 we can see that when the trajectory goes beyond the effective radiuses of the sources; a sharp increase in the function $q_s(b)$ is observed.

7. Conclusion. This article presents the probabilistic description of the detection of the moving objects in circular-symmetrical fields and non-symmetrical fields of the contiguous type sources and sources with finite scope. The concept of characteristic probability function of the source is introduced; this allow us to calculate the probability of successful passing of a random trajectory in the given source field. The opportunity of using the characteristic probability function for additional accounting of repeller sources on the trafficability maps for further optimization of the trajectory using the special algorithms is shown.

An iterative procedure has been developed that enables to find a piecewise-linear trajectory for which the probability of passing takes a given value, with restrictions on the allowable deviation from the original trajectory. When the boundary points of the trajectory are fixed, an analytical limit is found to which the function of the characteristic probability of passage tends with an unlimited increase in the iteration number. This limit can be used both for direct assessment of the upper boundary of the achievable probability of passing and for synthesis of the method of trajectory optimization with the moving endpoints.

The procedure developed in this article is much simpler in software implementation compared to more accurate methods for finding the optimum associated with the problem of solving systems of nonlinear equations and finding all their roots.

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ОПТИМИЗАЦИЯ ДВИЖЕНИЯ МОБИЛЬНОГО РОБОТА НА ПЛОСКОСТИ В ПОЛЕ КОНЕЧНОГО ЧИСЛА ИСТОЧНИКОВ-РЕПЕЛЛЕРОВ

Костюков В.А., Медведев М.Ю., В.Х. Пшихопов **Оптимизация движения мобильного робота на плоскости в поле конечного числа источников-репеллеров.**

Аннотация. Рассматривается задача планирования движения мобильного робота в конфликтной среде, которая характеризуется наличием областей, препятствующих выполнению роботом поставленных задач. Дается обзор основных результатов планирования пути в конфликтных средах. Отдельное внимание уделяется подходам, основывающимся на функциях рисков и вероятностных методах. Рассматриваются конфликтные области, которые формируются точечными источниками, генерирующими в общем случае несимметричные поля непрерывного типа. Предлагается вероятностное описание таких полей, примерами которых являются вероятность обнаружения или поражения мобильного робота. В качестве характеристики поля вводится понятие характерной вероятности функции источника, которая позволяет оптимизировать движение робота в конфликтной среде. Показана связь характерной вероятности функции источника и функции риска, которая может быть использована для постановки и решения упрощенных оптимизационных задач. Разрабатывается алгоритм планирования пути мобильного робота, обеспечивающий заданную вероятность прохождения конфликтной среды. Получена верхняя оценка вероятности прохождения заданной среды при фиксированных граничных условиях. Предложена процедура оптимизации пути робота в конфликтной среде, которая характеризуется более высокой вычислительной эффективностью, достигаемой за счет ухода от поиска точного оптимального решения к субоптимальному. Предложенные алгоритмы реализованы в виде программного обеспечения симулятора группы наземных роботов и исследуются методами численного моделирования.

Ключевые слова: планирование пути, конфликтная среда, характерная вероятностная функция, оптимизация движения.

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