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OBSERVATION IMPACT ASSESSMENT ON THE PREDICTION OF THE EARTH SYSTEM DYNAMICS USING THE ADJOINT-BASED METHOD

Steinle P., Tingwell C., Soldatenko S.A. **Observation Impact Assessment on the Prediction of the Earth System Dynamics Using the Adjoint-Based Method.**

Abstract. Mathematical models of the Earth system and its components represent one of the most powerful and effective instruments applied to explore the Earth system's behaviour in the past and present, and to predict its future state considering external influence. These models are critically reliant on a large number of various observations (in situ and remotely sensed) since the prediction accuracy is determined by, amongst other things, the accuracy of the initial state of the system in question, which, in turn, is defined by observational data provided by many different instrument types. The development of an observing network is very costly, hence the estimation of the effectiveness of existing observation network and the design of a prospective one, is very important. The objectives of this paper are (1) to present the adjoint-based approach that allows us to estimate the impact of various observations on the accuracy of prediction of the Earth system and its components, and (2) to illustrate the application of this approach to two coupled low-order chaotic dynamical systems and to the ACCESS (Australian Community Climate and Earth System Simulator) global model used operationally in the Australian Bureau of Meteorology. The results of numerical experiments show that by using the adjoint-based method it is possible to rank the observations by the degree of their importance and also to estimate the influence of target observations on the quality of predictions.

Keywords: variational data assimilation, adjoint model, forecast sensitivity, observation impact, Earth system.

1. Introduction. Mathematical models of the Earth system and its components such as the atmosphere, ocean, hydrosphere and biosphere, represent one of the most powerful and effective instruments applied to explore the Earth system's behaviour in the past and present, and to predict its future state considering external influence (e.g. [1-4] and references herein). These models include and parametrically describe numerous physical, chemical and biological processes and cycles such as water cycle, carbon and nitrogen cycles etc. Prediction of the Earth system dynamics under the influence of natural forcing and anthropogenic interventions represents one of the challenging issues of modern science. From the standpoint of dynamical systems theory, the Earth system consists of several interactive dynamical subsystems. Each of them covers a broad space-time spectrum of motions and a wide variety of physical and chemical processes. The Earth system components have specific physical, chemical and dynamical properties, unique structure and behaviour. They are closely related to each other via fluxes of energy, matter, water, aerosols, carbon dioxide and other chemical substances. Modern Earth system models are highly complex and resource intensive. These models, which can range substantially in their complexity, can be a simple concept or a set of partial differential equations that can be solved numerically by high-performance computers. Formally, the Earth system (or its any component) can be considered as dynamical system generated by the following vector-valued evolutionary differential equation:

$$dx/dt = \mathcal{L}\left(x(t), \alpha\right); \tag{1}$$

$$x\big|_{t=0} = x_0 , \tag{2}$$

where $\mathcal{L}$ is a nonlinear differential operator, x is a state vector, x_0 is a given vector-valued function defining the initial state of a system, and α is a vector of parameters.

Since equation (1) is solved numerically, it should be transformed to the discretised form. Equation (1) discretised on the model space-time grid can be written in the following compact from:

$$x_{k+1} = \mathcal{M}_{k,k+1}\left(x_k\right) + \varepsilon_k , \tag{3}$$

where $x_k \in \mathbb{R}^n$ is the n-dimensional state vector at time t_k representing the complete set of variables that determine the internal state of a system in question, $\mathcal{M}_{k,k+1} : \mathbb{R}^n \to \mathbb{R}^n$ is a discrete nonlinear operator that propagates the state variables from time t_k to time t_{k+1}, and $\varepsilon_k \in \mathbb{R}^n$ is model errors. Note that the model discrete operator indirectly includes known model parameters. It is usually assumed that the model (3) is "perfect" ($\varepsilon_k = 0$), i.e. given the initial condition x_0, equation (3) uniquely specify the path of dynamical system in its phase space.

Numerical models used in Earth system simulations are critically reliant on large amounts of Earth observation data that are required to correctly define the initial conditions through the process known as data assimilation (DA) (e.g. [5, 6]). As the practice shows, the quality of prediction is strongly affected by the observations – their volume, temporal and special distribution, and accuracy of measurements. In many applications, to simulate and predict the long-time behaviour of dynamical system (e.g. in climate studies) observation data are used to adjust a predictive model trajectory to newly obtained observations (see Figure 1; this figure was created based on the ideas discussed in [7]). To date DA remains one of the key issues in geophysical sciences. The basic goal of DA is to merge observations of any type with certain prior information which needs to be estimated in some way. For example, this prior information referred to as the background can be estimated by models used in prediction.

One of the most popular and effective DA methods is four-dimensional DA (4D-Var). In general terms, 4D-Var DA aims to define the initial state of a dynamical system in question by combining (in statistically optimal manner) the observations of state variables of a real physical system together with a background. 4D-Var procedures are mathematically formulated as an optimization problem, in which the initial condition plays the role of control vector and model equations are considered as constraints. The theoretical foundations of the study and the solution of such problems were laid in the classical works of R.E. Bellman [8], L.S. Pontryagin et al. [9], J.-L. Lions [10], G.I. Marchuk [11]. The variational approach was first used in the prediction of atmospheric processes by Sasaki [12] and then, starting from famous research papers [13-15], has been extensively explored in a vast number of publications.

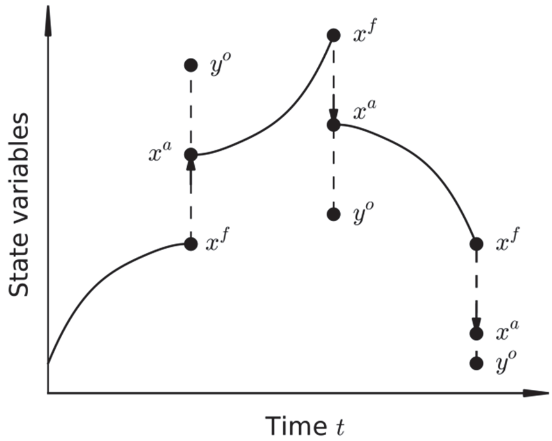

Fig. 1. The scheme of model trajectory adjustment to new observations

The ACCESS (Australian Community Climate and Earth System Simulator) at the Bureau of Meteorology [16] utilizes the 4D-Var scheme in incremental formulation developed at the UK Met Office [19]. The general idea of 4D-Var approach can be simply illustrated as follows. Suppose that at a certain initial time the background state x^b and some physical quantities y^{o} measured by instruments are known. Then [5]

$$x = x^b + \varepsilon^b, \quad y^{\mathrm{o}} = \mathcal{H}\left(x\right) + \varepsilon^{\mathrm{o}}, \tag{4}$$

where $\mathcal{H}$ is the (nonlinear) projection operator, that maps the space of model state into the space of observations, ε^b and ε^{o} are the errors of the background and observations respectively. Within the framework of 4D-Var, the initial state x_0 is estimated via the following optimization problem [5]:

$$x_0^a = \arg\min \mathscr{J}\left(x_0\right), \tag{5}$$

$$\mathscr{J}\left(x_0\right) = \frac{1}{2}\left\| x_0 - x_0^b \right\|_{B^{-1}}^2 + \frac{1}{2}\left\| \mathcal{H}\left(x\right) - y^o \right\|_{R^{-1}}^2, \tag{6}$$

where B and R are the error covariance matrices of the background and observations, respectively, $\| \cdot \|_A$ is the inner product with respect to the A matrix metrics, i.e. $\| a \|_{A^{-1}}^2 = a^T A^{-1} a$.

The cost (objective) function (6) is interpreted as follows. The first term that is the background term represents the deviation between the model initial state x_0 and the background x_0^b and calculated in the Euclidean norm L^2 described by the background covariance matrix B. The second term, the observation term, measures the deviation between observations y^o and the "model equivalent" of observations $\mathcal{H}\left(x\right)$. This term is calculated in the L^2 norm described by the observation-error covariance matrix R and is summed over the assimilation window.

The 4D-var problem is simply a minimization problem with constraints on x given by the model equation (3). If the observation operator is linear, we obtain a quadratic problem whose unique solution is provided by the Best Linear Unbiased Estimator (BLUE) [5]:

$$x_0^a = x_0^b + Kd, \tag{7}$$

where $K = \left(B^{-1} + H^T R^{-1} H\right)^{-1} H^T R^{-1}$ is the Kalman gain matrix, H is a linearized observation operator, and $d = y^o - Hx^b$ is the innovation vector.

When the observation operator is nonlinear, the variational data assimilation system considers a series of state variables x_j along which the nonlinear operator $\mathcal{H}$ can be linearized. This approach known as an incremental variational data assimilation was introduced in [18]. The first state variable is taken as the background state $x_0 = x^b$, and at iteration j the objective function is:

$$\mathscr{J}\left(\delta x_0\right) = \frac{1}{2}\left\| \delta x_0 - b_j \right\|_{B^{-1}}^2 + \frac{1}{2}\left\| H_j\left(\delta x\right) - d_j \right\|_{R^{-1}}^2, \tag{8}$$

where $\delta x_0 = x_{0,j} - x_{0,j-1}$ is the output result of the minimization, $\mathrm{b}_j = x_0^b - x_{0,j-1}$, $\mathrm{d}_j = y^{\mathrm{o}} - \mathrm{H}(x_{j-1})$, and H_j is the observation operator linearized around the state estimate x_{j-1}. To achieve the absolute minimum (not the local one), the first guess should be close enough to the truth.

The essential component of 4D-Var system is an adjoint model which, as will be shown below, plays a major role in the exploration of model forecast sensitivity to observations and in the assessment of observation impact on the accuracy of prediction of the Earth system and its components. We would like to emphasize that significant contribution to the theory of adjoint equations was made by G.I. Marchuk and his scientific school (e.g. [11, 19, 20]).

Commonly, the impact of observations on the prediction skill of Earth system models is evaluated by executing the so-called Observing System Experiment (OSE), also known as a Data Denial Experiment (DDE). In a DDE, the forecast skill of two individual runs are compared—one with all observations assimilated and the other with a given observation type (or individual instrument) withheld or added (e.g. [21]). Any change in the forecast accuracy is referred to the observations, which have been withheld. The approach can also be used to assess the impact of target or newly available observations. DDEs can be very helpful but come with disadvantages: they are computationally expensive and not suited to assess the impact of a single station in an observing network or individual measurement device. In addition, DDEs only provide information on the dataset that was withheld, and no information on the value of other subsets of observations.

Another technique, which is able to calculate the individual impact that each assimilated observation has, and is capable to continually generate and aggregate forecast impacts for all observations, was suggested in [22, 23]. This approach makes use of the adjoint models utilized within 4D-Var systems. The observation impact is measured by the reduction in the forecast error expressed as a total "moist" or "dry" energy norm. This method was subsequently implemented in several research and operational centres (e.g. [24, 25]). It is important that such a method uses the same computer code as 4D-Var systems.

This paper aims to illustrate the application of the adjoint-based approach to two coupled low-order chaotic dynamical systems and to the ACCESS global model. We emphasize that this technique is a powerful instrument that allows for not only evaluating the current observing network but also assessing the value of network components which will be used in the future, and, therefore, solve the problems of designing an observing network.

Low-order chaotic dynamical systems considered in this paper, represent computational tools which can be helpful for exploring various aspects of numerical modeling and predicting the behaviour of complex dynamical systems arising in geophysical, environmental, biological, engineering and other branches of science. For these models, the computational cost is insignificant. Consequently, they can be viewed as testing tools to mimic the behaviour of complex systems and, in particular, to explore the forecast sensitivity with respect to observations.

2. Method. As mentioned above, the simplest, but computationally expensive method to assess the impact of observations coming from various sources is the OSE. The main idea behind this method is as follows. Suppose we calculate the forecast (the future state of dynamical system in question) by integrating the model equations over a given time interval $[t_0, t^f]$, where t^f is a verification time (the time at which the forecast accuracy is assessed). Initial conditions for this experiment are determined through 4D-Var utilizing all types of observations. Assume that the forecast accuracy is verified by the use some quantitative measure E_f. Then we integrate the model equation utilizing via 4D-Var all types of observations excluding y_s^{o}. For this run the forecast accuracy is characterized by E_b. The difference $E_f - E_b$ quantifies the impact of observations y_s^{o} on the forecast accuracy. However, this approach is computationally ineffective and inconvenient to assess the impact of observations of different types and individual measurements.

Meanwhile, using the adjoint-based technique we can assess the impact of any or all available observations in a computationally efficient way. This method is very appropriate since adjoint models are embedded in 4D-Var systems. Observation impact is computed using (a) sensitivity functions which are components of the adjoint sensitivity gradient of some cost function that characterizes the forecast error, and (b) innovations $y^o - \mathcal{H}\left(x^b\right)$ [23].

Let $\mathcal{R}$ be a scalar response function which is dependent on the system state variables at verification time $t^f : \mathcal{R} = \mathcal{R}\left(x^f\right)$. From the Taylor expansion we can derive the first-order variation of $\mathcal{R}$ at time t^f:

$$\delta\mathcal{R} = \left\langle \delta x^f, \partial\mathcal{R}/\partial x^f \right\rangle = \left\langle \mathrm{M}x_0, \partial\mathcal{R}/\partial x^f \right\rangle. \quad (9)$$

Here $\langle\ ,\ \rangle$ denotes the dot product, and the forecast variation is expresses via tangent linear model: $\delta x^f = \mathrm{M}\delta x_0$, where M is a linearized model operator. Let M^* be the adjoint of M such that $\langle \mathrm{M}x, y \rangle = \langle x, \mathrm{M}^* y \rangle$.

Since the adjoint of a real matrix equals to its transpose then $\mathrm{M}^* = \mathrm{M}^\mathrm{T}$ and the equation (9) takes the form [23]:

$$\delta\mathcal{R} = \left\langle \delta x_0, \mathrm{M}^\mathrm{T} (\partial\mathcal{R} / \partial x^f) \right\rangle; \tag{10}$$

and the sensitivity of response function to the initial state can be expressed as [23]:

$$\frac{\partial\mathcal{R}}{\partial x_0} = \mathrm{M}^\mathrm{T} \frac{\partial\mathcal{R}}{\partial x^f}. \tag{11}$$

Thus, running the adjoint model backward in time with the sensitivity of $\mathcal{R}$ at the verification time as input, one can calculate the sensitivity of $\mathcal{R}$ with respect to the initial conditions. Generally, any differentiable scalar function that represents the forecast accuracy can be considered as the response function (e.g. single model variable or some function of model state variables). Commonly, the forecast error relative to the "true" state x^t is measured in terms of the "total energy norm" [26]:

$$E = (x^f - x^t)^\mathrm{T} \mathrm{C} (x^f - x^t), \tag{12}$$

where C is a diagonal matrix of weighting coefficients. The sensitivity of E with respect to initial conditions is expressed as [27]:

$$\frac{\partial E}{\partial x_0} = 2\mathrm{M}^\mathrm{T} C (x^f - x^t). \tag{13}$$

At some initial time t_0, there are two state estimations: x_0^a, which is obtained using 4D-Var, and x_0^b, which is obtained via previous model run. Thus, two forecast errors, E_f and E_b, can be defined as [23]:

$$E_f = (x_a^f - x^t)^\mathrm{T} \mathrm{C} (x_a^f - x^t), \quad E_b = (x_b^f - x^t)^\mathrm{T} \mathrm{C} (x_b^f - x^t), \tag{14}$$

where x_a^f and x_b^f are the predicted states initiated from x_0^a and x_0^b.

To estimate the impact of observations on the forecast error reduction, the response function can be defined as the difference between E_f and E_b:

$$\delta\mathcal{R} \equiv \frac{1}{2}\Delta E = \frac{1}{2}\left(E_f - E_b\right). \tag{15}$$

The linear approximation of error reduction $\delta E \approx \Delta E$ is given by [23]:

$$\delta E = \left\langle \left(y^o - \mathrm{H}\left(x_0^b \right) \right),\ \frac{1}{2}\mathrm{K}^{\mathrm{T}} \left(\frac{\partial E_f}{\partial x^a} + \frac{\partial E_b}{\partial x^b} \right) \right\rangle, \quad (16)$$

where K^{T} is a transpose of the Kalman gain matrix.

The equation (16) gives the estimate of the forecast error reduction δE produced by any or all observations. Figure 2 (adopted from [23]) shows the schematic representation of the discussed approach for evaluating the forecast sensitivity with respect to observations and assessing the impact of various observations on the forecast accuracy. To estimate the impact of all types of observations we only need to perform a single system's run.

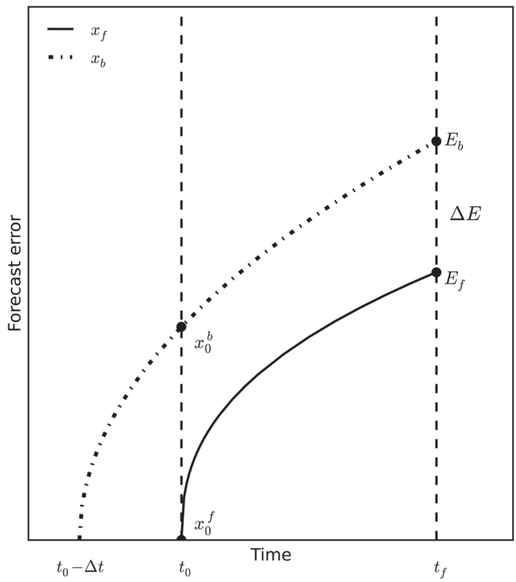

Fig. 2. Schematic representation of the adjoint-based method for observation impact assessment

It is obvious that if the assimilated observations improve the forecast accuracy at the verification time x^f, then the forecast error δE is reduced, and the value δE will be negative. However, if the assimilated observations diminish the forecast quality, the value δE will be positive.

3. Results. For illustrative purposes only we first apply the method discussed in Section 2 to estimate the observation impact on the prediction of dynamics of coupled chaotic dynamical system [27] described in the appendix. This model has six state variables. The following information should be available to solve the problem: the “true” trajectory of a system x^t, the background (the first guess) trajectory x^b, and observations y^{o}.

In our calculations, we used synthetic data. The "true" trajectory is obtained by integrating model equations numerically with the initial conditions x_0^t taken on the system’s attractor. The background (the first guess) trajectory x^b is obtained by integration of the system equations with predefined initial conditions x_0^b which is specified as $x_0^b = x_0^t + \delta^b$, where δ^b is a normally distributed random perturbation with a standard deviation of σ_b applied to all elements of the state vector.

To take into account the background errors, the assumption $\mathrm{B}_0 = \sigma_b^2 \mathrm{I}$ is used, where I is the identity matrix. We assume that the value of $\sigma_b = 0.2$ is applied to all elements of the state vector. Observations y^{o} are defined for every $2\Delta t$ within the assimilation window, which has a total temporal length of $50\Delta t$. The observed values are generated by adding Gaussian random noise with zero mean and specified standard deviation σ_{o} to the true state x^t. In calculation we assume that $\sigma_{\mathrm{o}}^{(1)} = 0.05$ ("accurate" observations) and $\sigma_{\mathrm{o}}^{(2)} = 0.1$ ("inaccurate" observations) for "fast" variables, and $\sigma_{\mathrm{o}}^{(1)} = 0.1$ and $\sigma_{\mathrm{o}}^{(2)} = 0.2$ for "slow" variables.

Since observation grid and model grid are the same, the linearized observation operator is simply an identity mapping $\mathrm{H} \equiv 1$. Under the assumption that observation errors are the same for all variables, the observation covariance matrices are defined as $\mathrm{R}_k = \mathrm{R} = \sigma_{\mathrm{o}}^2 \mathrm{I}$.

To minimize the objective function, the conjugate gradient method, resulting in the analysis x_0^a, has been applied. The forecast trajectory is then obtained by integrating the model equations given initial conditions x_0^a.

To estimate the prediction accuracy and the reduction of the forecast error due to observations we use the relative error in energy norm:

$$E_r = \left[(x^t - x^f)^{\mathrm{T}} (x^t - x^f) \big/ (x^t)^{\mathrm{T}} x^t \right]^{\frac{1}{2}}. \tag{17}$$

The impact of observations is assessed using the ensemble of trajectories generated by randomly produced initial conditions. Table 1 shows the relative error reductions averaged over 500 ensemble members for both "accurate" and "inaccurate" observations. In this table, the forecast errors are computed for different verification time t^f. The coupled model used in these experiments is chaotic, therefore, its behaviour is highly sensitive to initial conditions. Thus, the forecast accuracy strongly depends on how accurately we can specify the initial state of the dynamical system in question. In turn, the accuracy of initial conditions depends on available observations, the model used in producing forecasts, and data assimilation system. In numerical experiments, the accuracy of observations is specified by the standard deviation σ_o (see above).

Table 1. Observation impact estimates for different verification times for "accurate" $(\delta E_r^{(1)})$ and "inaccurate" $(\delta E_r^{(2)})$ initial conditions

	Verification time			
	0.5	1.0	1.5	2.0
$(\delta E_r^{(1)})$	-0.91	-2.33	-4.32	-3.27
$(\delta E_r^{(2)})$	-0.58	-1.74	-2.53	-1.22

Table 1 illustrates that both "accurate" and "inaccurate" observations show positive impact on the forecast accuracy since the relative energy norm reductions are negative. The impact of “accurate” observations” is, however, larger than the impact of “inaccurate” observations. It is important that the observation impact estimate δE_r is valid over a limited lead time t_{lim}^f since the adjoint model used in calculation of δE_r is derived from a linear forward propagation model known as a tangent linear model. Numerical experiments shown that $t_{lim}^f \approx 2.2$ of dimensionless time units.

For coupled chaotic dynamical system developed on the bases of model [28], the prediction error reductions by observations computed for different verification times t^f and "accurate" observations are presented in Table 2.

Table 2. Observation impact estimates for different verification times for "accurate" initial conditions

	Verification time			
	1	2	3	4
δE_r	-4.53	-3.39	-2.34	-0.39

This table shows that the shorter the forecast range the larger the error reduction or, in other words, the prediction accuracy. These results were obtained by ensemble simulations with 500 ensemble members. For reference, the relative observation impacts (in percentage points) calculated for each observation variable at $t^f = 3$ are shown in Table 3. Observations of *z*-component provide the highest impact on the forecast error reduction while observations of *Y*-component the smallest impact.

Table 3. Relative observation impact (in percentage points) of each model variable for verification time of $t^f = 3$

Verification time				
x	y	z	X	Y
26.5	21.8	36.2	14.7	0.8

Let us now discuss some results obtained via ACCESS global model [16]. The model grid covers the globe with a horizontal resolution of N512 (1024×769 grid points along longitude and latitude, respectively, with average distance between grid points about 25 km), with 70 vertical levels up to ~80 km altitude. The linear perturbation forecast model (a tangent linear model with moist physics) and its adjoint used in 4D-Var and forecast-to-observations experiments has the same vertical resolution as the nonlinear model with a horizontal resolution of N215 (about 60 km).

In the Bureau of Meteorology, a total of 40 million observations are processed daily. Most of these data are satellite measurements. However, only about 10 percent of all observations (~ 4 million) are used in the assimilation system to calculate the initial conditions for the global ACCESS prediction. The following is a summary of the observation types assimilated in the ACCESS 4D-Var global system:

– Surface observations: SYNOP (synoptic network weather stations), SHIP (ship-based instruments), WINPRO (wind profilers), DRIBU (buoy-based instruments) ;

– Upper air observations: TEMP (radiosondes), PILOT (wind observations from pilot balloons and radar profilers), aircraft reports (AIREPS, AMDARS);

– Satellite winds: scatterometer surface winds (ASCAT), atmospheric vector winds (AMV);

– Microwave radiances: ATOVS (AMSU A, B and MHS);

– Infrared radiances: ATOVS (HIRS), AIRS);

– Infrared atmospheric sounding interferometer (IASI);

– Cross-track infrared sounder (CrIS), microwave humidity sounder (MHS), atmospheric infrared sounder (AITS); advanced technology microwave sounder (ATMS);

– Satellites and occultation data from various global navigation satellite systems (GNSS) such as the Global Positioning System;

– Geostationary operational environmental satellite system (GEOS).

The analysis (initial conditions for the prediction model) is generated through the 4D-Var system with a 6-h assimilation window. Observation impacts represent an estimate of the change in a 24-h forecast error as a consequence of the assimilation of observations. Forecast error is measured in terms of a moist energy norm calculated from the surface to the 150 hPa level over the globe and the northern and southern hemispheres. The adjoint-based observation impacts were calculated from 00Z 1 January 2017 to 00Z 31 December 2017 in 6-h intervals. The experiment details are summarized in Table 4. The total energy norm used to calculate the forecast error reduction due to observations is defined as follows [24]:

$$E = \delta x^{\mathrm{T}} \mathrm{C} \delta x = \frac{1}{M_D} \iiint A r^2 \cos\varphi d\Sigma d\eta \ , \tag{18}$$

$$A = \frac{1}{2}\left(\rho u'^2 + \rho v'^2 + \frac{\rho g^2}{\theta^2 N^2}\theta'^2 + \frac{1}{\rho c^2} p'^2 + \varepsilon \frac{\rho L^2}{c_p} q'^2 \right), \tag{19}$$

where M_D is the mass of the atmosphere in the integration domain D; u, v are the zonal and meridional wind components, respectively; θ is the reference potential temperature, p is pressure; q is the specific humidity; c_p is the heat capacity at constant pressure; L is the latent heat of water vapor condensation; g is the acceleration of gravity; ρ is the air density; c is the speed of sound; N^2 is the square of the Brunt-Väisälä frequency; Σ is the horizontal domain of integration defined by the local projection matrix and η is the vertical coordinate. The primed variables denote the components of linear perturbation forecast model.

Table 4. Summary of experiment

Data period	From 00Z 1 January 2016 to 00Z 31 December 2016
Impact measure	24-hour forecast error reduction on the moist energy norm calculated from the surface to 150 hPa level over the globe and the northern and southern hemispheres.
Prediction system	Operational version of the Bureau of Meteorology prediction system with the resolution of N512 for the forecasting model and N216 for the inner loop of 4D-Var, in horizontal, and 70 levels in vertical. The adjoint of perturbation forecast model includes the moist physics.

The procedure of observation impact assessment is a post-processing routine. For each analysis time (four times per day), the adjoint forecast-to-observation system produces an ASCII output file that contains the information on all the observations including satellite and in situ, non-satellite observation types. The following information is contained in the ASCII output file:

- Sequential number of observation;
- The observation value;

– The value of innovation (the difference between observation and background values);

– Sensitivity of the forecast to observation;

– Latitude and longitude of observation;

– Pressure level of observation;

– Identifier of the instrument type (radiosonde, surface station, wind profiler etc.);

– Identifier of the observation variable type (temperature, moisture, pressure, horizontal wind components etc.);

– The time offset of the observation from the analysis time;

– Observation error variance;

– WMO (World Meteorological Organization) station identification number;

– Satellite identifier, satellite instrument and channel number.

Calculation of the observation impact is a multi-step process. At each analysis time, the ASCII output file is processed into a set of JSON (Java Script Object Notation) files, from which a set of Python-based tools aggregate the individual forecast sensitivities based on observation type and/or station and statistically analyze and visualize the results.

The calculated average observation impact per day of each type of observation on the global forecast is illustrated in Figure 3. This figure shows that all subsets of observations are beneficial, i.e. the impact measure is negative. That is, each individual observation type leads to the reduction of the forecast error of the total energy norm. The most significant observation impact is demonstrated by the Infrared Atmospheric Sounding Interferometer (IASI). Sonde (TEMP) data has the second largest impact. High impact is also provided by AMSUA (microwave sounder radiances), JPSSO-CrIS (cross track infrared sounders), Aircraft and SYNOP (surface observations at land stations). However, MTSAT (atmospheric motion retrievals), HIRS (infrared sounder radiances) and WINPRO (wind profilers) show very small impact. The small impact of WINPRO likely results from the cessation of NOAA's supply of the wind profiler data to the Global Observing System. Currently, only European (CWINDE, 29 wind profilers), Japanese (WINDAS, 31 wind profilers) and Australian (11 wind profilers) wind profiler networks serve as a source of observations. The European and Japanese profilers are operating in densely observed areas of the world, and so the low impact is not surprising. In addition, there are about 50 standalone wind profilers around the globe, which provide data via the Global Observing System.

The volume of satellite information is about 90% of the volume of meteorological information processed via the 4D-Var system. Consequently, the impact of satellite data is always high. However, the analysis of impact per observations for different observation subsets shows (see Figure 4) that the most substantial contributions into the reduction of forecast error per one observation demonstrate PILOT, BUOY and satellite data such as ESA and GOES.

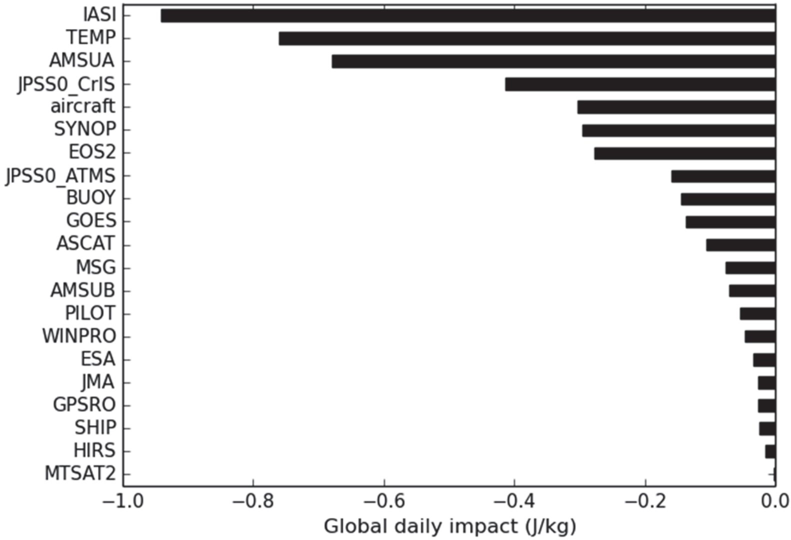

Fig. 3. Average observation impact per day (J/kg) over the globe of each type of observation

The spatial distribution of observation sources is inhomogeneous over the globe. In the northern hemisphere the ratio of land to ocean is about 1 to 1.5. In contrast, in the southern hemisphere the fraction of ocean is about 80% while the land fraction is only about 20%. Consequently, the network of synoptic and aerological stations (upper air observations) in the northern hemisphere is significantly denser than in the southern hemisphere, and the number of synoptic stations and aerological stations in the northern hemisphere significantly exceeds the corresponding number of stations in the southern hemisphere. Figures 5 and 6 illustrate the contribution of observations from both northern and southern hemispheres to the reduction of global forecast error. Sonde observations, Aircraft data, IASI and AMSUA obtained in the northern hemisphere demonstrate the most important impact on the forecast quality (skill). In the southern hemisphere, data from IASI and AMSUA contribute the greatest to the forecast error reduction. Satellite data such as MTSAT2, EOS2 and HIRS are less influential in terms of influence on the forecast accuracy.

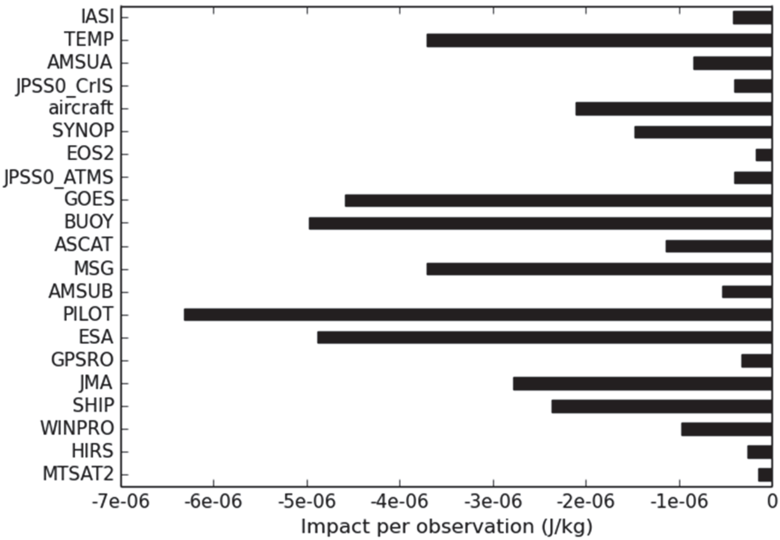

Fig. 4. Average impact per observation (J/kg) for the global forecast

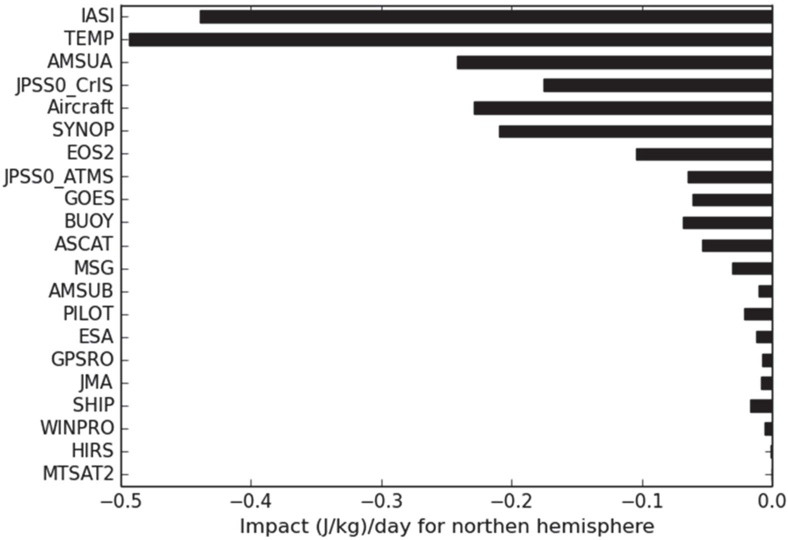

Fig. 5. Daily observation impact (J/kg) for the northern hemisphere calculated for the period January – December 2017

Results obtained show that all observation types positively influence the forecast accuracy both over the globe and northern and southern hemispheres. However, satellite data play a crucial role in weather prediction in the southern hemisphere. We should keep in mind that the adjoint-based method used in calculations is restricted by the linearity of the algorithm (the perturbation forecast model is linear and, certainly, its adjoint is also linear), which makes it valid only to evaluate short-range forecasts (0 to 48 hours).

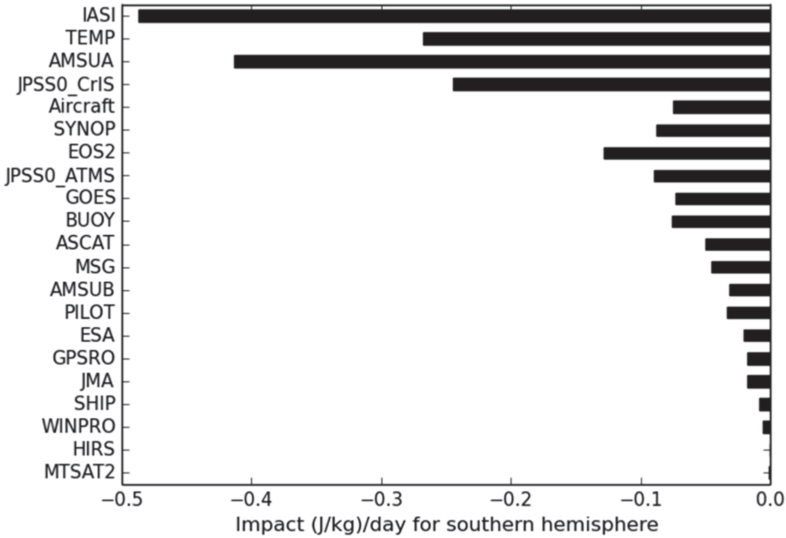

Fig. 6. Daily observation impact (J/kg) for the southern hemisphere calculated for the period January – December 2017

4. Discussion and conclusion. In this paper, we evaluated impacts of various types of observations, in situ observations (ground and ocean-based synoptic and ship observations, wind profiler information, radiosonde and wind balloon upper air observations, and aircraft data) and satellite information, on the accuracy of short-term prediction of global weather conditions using the adjoint-based approach embedded into the ACCESS and its 4D-Var. The impact is measured by the reduction in the 24-hour forecast error expressed as a moist energy norm calculated globally. Overall, all observation types have a positive impact on the forecast accuracy.

Using the adjoint-based method we can support the decision-making process regarding the evolution of the observing network. This powerful technique represents a tool for guiding the design and running of an efficient and effective observing network at the national level and internationally. Future studies are required to explore the seasonal and inter-annual variability of observation impacts.

We also presented two coupled chaotic dynamical systems developed on the bases of the original Lorenz chaotic models [27, 28] which can imitate the essential features of natural, societal and technical dynamical systems that possess the deterministic chaos. These two coupled systems together with variational assimilation subsystems provide the computational framework for testing different numerical methods and algorithms, and estimating the observation impacts on the prediction of chaotic dynamics.

The Bureau of Meteorology has been applying an adjoint-based approach to estimate the impact of observations on forecasts for about five years. The results obtained are consistent with the available estimates calculated in different research and prediction centres around the world (e.g. [29 – 35]).

5. Appendix. In this appendix we describe two low-order coupled chaotic dynamical systems used in this study. We start from the model obtained by coupling of two ("fast" and "slow") versions of the original Lorenz system [27] (from hereon, L63) with specific time scales differing by a factor ε. We emphasize that L63 is deterministic, however, describes the phenomenon known as "deterministic chaos": over time the behaviour of a simulated system begins to resemble a random process, even though the system is defined by deterministic laws and described by deterministic equations. This phenomenon was first uncovered by Lorenz as he observed the sensitive dependence of atmospheric convection model output on initial conditions [27].

The L63 system is derived by strong spectral truncation of Saltzman's equations, which describe the Rayleigh-Benard convection, and consists of three autonomous ordinary differential equations for time-dependent variables x, y, and z: with x corresponding to the intensity of the convective motion in terms of the stream function, y to the temperature differences between rising and descending currents, and z to the departure of the vertical temperature gradient from its equilibrium magnitude. The L63 contains three positive parameters σ, r, and b, with σ being the Prandtl number, r the normalized Rayleigh number, and b a geometric parameter characterizing length scale of the convective cell.

The L63 can imitate some essential properties of the general circulation of the atmosphere and ocean since the heat flux from equator to the poles can be represented by variable z, which is proportional to meridional temperature gradient that can be represented by parameter r.

A coupled nonlinear chaotic dynamical system is represented by the following set of autonomous differential equations [36]:

a) The "fast" subsystem:

$$\dot{x} = \sigma\left(y - x\right) - c\left(aX + k\right),$$

$$\dot{y} = rx - y - xz + c(aY + k),$$

$$\dot{z} = xy - bz + c_z Z.$$

b) The “slow” subsystem:

$$\dot{X} = \varepsilon\sigma\left(Y - X\right) - c\left(x + k\right),$$

$$\dot{Y} = \varepsilon(rX - Y - aXZ) + c(y + k),$$

$$\dot{Z} = \varepsilon(aXY - bZ) - c_z z,$$

where the lower-case letters x, y and z represent the state variables of the "fast" model, upper-case letters X, Y and Z denote the state variables of the "slow" model, $\sigma > 0$, $r > 0$, and $b > 0$ are the parameters of the original L63 model, ε is a time-scale factor (e.g. if $\varepsilon = 0.1$, then the "slow" subsystem is ten times slower than the "fast" subsystem), c is a coupling strength parameter for x, X, y, and Y variables, c_z is a coupling strength parameter for z and Z variables, k is a “decentring” parameter, and a is a parameter representing the amplitude scale factor ($a = 1$ indicates that "slow" and "fast" subsystems have the same amplitude scale).The coupling strength parameters c and c_z control the interconnection between "fast" and "slow" subsystems: the smaller the parameters c and c_z, the weaker the interdependence between two subsystems.

Essential properties of this chaotic system have been considered in detail earlier [37]. It was underlined that the temporal dynamics of coupled L63 is strongly conditioned by its parameters. Standard values of the L63 parameters corresponding to chaotic behaviour are $\sigma = 10$, $b = 8/3$, and $r = 28$ [27]. These parameter values are used in this study since the motions in the atmosphere and ocean are inherently chaotic. Note that for $\sigma = 10$ and $b = 8/3$, there is a critical value for parameter r, equal to 24.74, and any r larger than 24.74 induces chaotic behaviour of the L63 system.

The time-scale factor ε, “decentring” parameter k and the amplitude scale factor a are taken to be 0.1, 0 and 1 respectively. Without loss of generality, we can assume that and $c = c_z$. The coupling strength c determines the strength of interactions between "fast" and "slow" subsystems and, therefore, the dynamics of the entire coupled system. In numerical experiments we assumed that the coupling between two subsystems is weak, therefore c=0.15 [37].

The system of model equations is numerically integrated by fourth order Runge-Kutta algorithm with a time step $\Delta t = 5 \times 10^{-3}$. To discard the initial transient period the numerical integration starts at time $t_{-\tau} = 2^{15}\Delta t$ with the initial conditions generated randomly around the point $x\left(-\tau\right) = \left(0.01; 0.01; 0.01; 0.02; 0.02; 0.02\right)^{\mathrm{T}}$ and finishes at time t=0. This guarantees that the calculated state vector $x_0 = x(0)$ is on the system’s attractor.

The state vector x_0 is then used as the initial conditions for further numerical experiments. Note that for $\Delta t = 5 \times 10^{-3}$, the numerical integration with length of 200 time steps corresponds to one dimensionless unit of time.

The next coupled chaotic dynamical system is also composed of "fast" and "slow" models. The "fast" model represents the chaotic dynamical system developed by Lorenz to study the large-scale atmospheric motions [28] (herein is referred to as L84), while the "slow" model, in the absence of coupling to the "fast" model, is a simple harmonic oscillator [38]. The following two sets of autonomous differential equations describe:

a) The "fast" model [28]:

$$\dot{x} = -y^2 - z^2 - ax + aF,$$

$$\dot{y} = xy - cy - bxz + G + aX,$$

$$\dot{z} = xz - cz + bxy + aY.$$

b) The "slow" model [38]:

$$\dot{X} = -\omega Y - \beta Y,$$

$$\dot{Y} = \omega X - \beta z,$$

where x is the intensity of the symmetric, globally averaged westerly wind current (equivalent to the meridional temperature gradient); y and z are the amplitudes of cosine and sine phases of a series of superposed large scale eddies, which transport heat poleward; F and G represent the thermal forcing terms due to the average north-south temperature contrast and the earth-sea temperature contrast, respectively, ω is the ocean oscillation frequency, X and Y are zonal asymmetries in sea surface temperature, which interact with the model atmosphere's eddy fields (y and z).

The "fast" model (the original model proposed by Lorenz) is a Galerkin truncation of the Navier-Stokes equations and gives the simplest approximation of the general circulation of the atmosphere. This model has been widely used in climatological studies and its properties have been explored extensively.

Values of the model parameters used in numerical experiments are as follows [45]: $a = 0.12$, $b = 4b$, $c = 0.5$, $F = 8$, $G = 0.25$, $\beta = \gamma = 0.1$, and $\omega = 2\pi\lambda / 4$, where $\lambda = 0.0274$.

The model equations are solved numerically by fourth order Runge-Kutta algorithm with a time step $\Delta t = 5 \times 10^{-3}$. The initial transient period is discarded, as was mentioned above for the L63 model.

References

1. Biermann F. Earth System Governance: World Politics in the Anthropocene. MIT Press. 2014. 267 p.
2. Hajima T. et al. Modeling in Earth system science up to and beyond IPCC AR5. *Progress in Earth and Planetary Science*. 2014. vol. 1. no. 1. 29 p.
3. Goose H. Climate system dynamics and modelling. Cambridge University Press. 2015. 273 p.
4. Gellelman A., Rood R.B. Demystifying climate models: a user's guide to Earth system models. Springer Nature. 2016. 274 p.
5. Kody L., Andrew S., Konstantinos Z. Data assimilation: A mathematical introduction. Springer. 2015. 242 p.
6. Fletcher S.J. Data assimilation for the geosciences: From theory to application. Elsevier. 2017. 908 p.
7. Leith C.E. Numerical models of weather and climate. *Plasma physics and controlled fusion*. 1993. vol. 35. 919 p.
8. Bellman R.E. Dreyfus S.E. Applied dynamic programming. Princeton University Press. 2015. vol. 2050. 392 p.
9. Pontryagin L.S., Boltyanskii V.G., Gamkrelidze R.V., Mishchenko E.F. The mathematical theory of optimal processes. WileyEnglish. 1962. 360 p.
10. Lions J.L. Control optimal des systemes gouvernes par des equations aux derivees Partielles. Dunod. 1968. 426 p.
11. Marchuk G.I. Numerical methods in weather prediction. Academic Press. 1974. 288 p.
12. Sasaki Y. Some basic formalism in numerical weather analysis. University of Oklahoma. 1970. vol. 98. pp. 875–883.
13. Penenko V.V., Obraztsov N.N. Variational method of adapting of meteorological fields. *Meteorology and Hydrology*. 1976. vol. 11. pp. 3–16.
14. Le Dimet F.-X., Talagrand O. Variational algorithms for analysis and assimilation of meteorological observations: theoretical aspects. *Tellus A*. 1986. vol. 38. no. 2. pp. 97–110.
15. Courtier P., Talagrand O. Variational assimilation of meteorological observations with the adjoint equations. Part 2: Numerical results. *Quarterly Journal of the Royal Meteorological Society*. 1987. vol. 113. no. 478. pp. 1329–1347.
16. Puri K. et al. Implementation of the initial ACCESS numerical weather prediction system. *Australian Meteorological and Oceanographic Journal*. 2013. vol. 63. pp. 265–284.
17. Rawlins F. et al. The Met Office global four-dimensional variational data assimilation scheme. *Quarterly Journal of the Royal Meteorological Society*. 2007. vol. 133. no. 623. pp. 347–362.
18. Courtier P., Thepaut J.-N., Hollingsworth A. A strategy for operational implementation of 4D-Var, using an incremental approach. *Quarterly Journal of the Royal Meteorological Society*. 1994. vol. 120. no. 519. pp. 1367–1387.
19. Marchuk G.I. Adjoint equations and analysis of complex systems. Kluver Academic Publishers. 1995. 468 p.
20. Marchuk G.I., Agoshkov V.I., Shutyaev V.P. Adjoint equations and perturbation theory. CRC Press. 1996. 288 p.
21. Kelly G., Thépaut J.-N. Evaluation of the impact of the space component of the Global Observing System through Observing System Experiments. ECMWF Newsletter. 2017. vol. 113. pp. 16–28.
22. Baker N., Daley R. Observation and background adjoint sensitivity in the adaptive observation targeting problem. *Quarterly Journal of the Royal Meteorological Society*. 2000. vol. 126. pp. 1434–1454.
23. Langland R.H., Baker N.L. Estimation of observation impact using the NRL atmospheric variational data assimilation adjoint system. *Tellus A: Dynamic Meteorology and Oceanography*. 2004. vol. 56. no. 3. pp. 189–201.
24. Lorenc A.C., Marriott R.T. Forecast sensitivity to observations in the Met Office Global numerical weather prediction system. *Quarterly Journal of the Royal Meteorological Society*. 2014. vol. 140. no. 678. pp. 209–224.

25. Soldatenko S., Tingwell C., Steinle P., Kelly-Gerreyn B.A. Assessing the impact of surface and upper-air observations on the forecast skill of the ACCESS numerical weather prediction model over Australia. *Atmosphere.* 2018. vol. 9. no. 1. 23 p.
26. Errico M.R. Interpretations of an adjoint-derived observational impact measure. *Tellus A: Dynamic Meteorology and Oceanography.* 2007. vol. 59. no. 2. pp. 273–276.
27. Lorenz E.N. Deterministic nonperiodic flow. *Journal of the Atmospheric Sciences.* 1963. vol. 20. pp. 130–141.
28. Lorenz E.N. Irregularity: A fundamental property of the atmosphere. *Tellus A.* 1984. vol. 36. no. 2. pp. 98–110.
29. Lupu C., Cardinali C., McNally A.P. Adjoint-based forecast sensitivity applied to observation error variances turning. *Quarterly Journal of the Royal Meteorological Society.* 2015. vol. 141. no. 693. pp. 3157–3165.
30. Cardinali C., Healy S. Impact of GPS radio occultation measurements in the ECMWF system using adjoint-based diagnostics. *Quarterly Journal of the Royal Meteorological Society.* 2014. vol. 140. no. 684. pp. 2315–2320.
31. English S. et al. Impact of satellite data. European Centre for Medium-Range Weather Forecasts. 2013. vol. 711. 48 p.
32. Jung B.-J., Kim H.M. Adjoint-derived observation impact using WRF in the Western North Pacific. *Monthly Weather Review.* 2013. vol. 141. no. 11. pp. 4080–4097.
33. Hoover B.T., Langland R.H. Forecast and observation-impact experiments in the Navy Global Environmental Model with assimilation of ECWMF Analysis Data in the global domain. *Journal of the Meteorological Society of Japan.* 2017. vol. 95. no. 6. pp. 369–389.
34. Cioaca A., Sandu A., de Sturler E. Efficient method for computing observation impact in 4D-Var data assimilation. *Computational Geosciences*. 2013. vol. 17. no. 6. pp. 975–990.
35. Janiskoca M., Cardinali C. On the impact of the diabatic component in the forecast sensitivity observation impact diagnostics. *Data Assimilation for Atmospheric, Oceanic and Hydrologic Applications.* 2017. vol. 3. pp. 483–511.
36. Sequeira L., Kirtman B. Predictability of a low-order interactive ensemble. *Nonlinear Processes in Geophysics*. 2012. vol. 19. no. 2. pp. 273–282.
37. Soldatenko S., Steinle P., Tingwell C., Chichkine D. Some aspects of sensitivity analysis in variational data assimilation for coupled dynamical systems. *Advances in Meteorology.* 2015. vol. 2015. 22 p.
38. Wittenberg A.T., Anderson J.L. Dynamical implications of prescribing part of a coupled system: results from a low-order model. *Nonlinear Processes in Geophysics*. 1998. vol. 5. no. 3. pp. 167–179.

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ОЦЕНКА ВЛИЯНИЯ НАБЛЮДЕНИЙ НА ПРОГНОЗИРОВАНИЕ ДИНАМИКИ ЗЕМНОЙ СИСТЕМЫ С ПОМОЩЬЮ МЕТОДА СОПРЯЖЕННЫХ УРАВНЕНИЙ

Стайнли П., Тингвелл К., Солдатенко С.А. **Оценка влияния наблюдений на прогнозирование динамики земной системы с помощью метода сопряженных уравнений.**

Аннотация. Математические модели земной системы служат мощным и эффективным инструментом, используемым для изучения поведения процессов, протекающих в сферических оболочках нашей планеты, в прошлом и настоящем, а также для прогнозирования их в будущем с учетом внешних воздействий. Качество моделирования и прогнозирования природных процессов с применением соответствующих математических моделей в значительной степени зависит от достоверности и объема информации, характеризующей состояние рассматриваемой физической системы (например, атмосферы) в некоторый начальный момент времени. Источниками этой информации служат различные стационарные и подвижные технические средства, интегрируемые в единую глобальную наблюдательную сеть. Поскольку развитие средств наблюдения дорогостоящее мероприятие, очень важно иметь возможность оценивать эффективность как существующей, так и планируемой наблюдательной сети. Цель настоящей работы состоит в том, чтобы, с одной стороны, рассмотреть подход, основанный на сопряженных уравнениях и позволяющий оценивать влияние различных наблюдений на точность прогнозирования эволюции основных компонентов земной системы (атмосферы и океана), и с другой стороны — проиллюстрировать применение этого подхода на примере двух хаотических малопараметрических динамических систем и глобальной модели ACCESS (моделирование австралийского климата и земной системы), используемой в Австралийском метеорологическом бюро. Результаты численных экспериментов демонстрируют высокие возможности метода сопряженных уравнений, который позволяет ранжировать измерительную информацию, получаемую с помощью различных технических средств, по степени ее важности, а также оценить влияние наблюдений на качество прогнозов.

Ключевые слова: вариационное усвоение информации, сопряженные уравнения, чувствительность прогноза к наблюдениям, земная система.

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Литература

1. *Biermann F.* Earth System Governance: World Politics in the Anthropocene // MIT Press. 2014. 267 p.
2. *Hajima T. et al.* Modeling in Earth system science up to and beyond IPCC AR5 // Progress in Earth and Planetary Science. 2014. vol. 1. no. 1. 29 p.
3. *Goose H.* Climate system dynamics and modelling // Cambridge University Press. 2015. 273 p.
4. *Gellelman A., Rood R.B.* Demystifying climate models: a user's guide to Earth system models // Springer Nature. 2016. 274 p.
5. *Kody L., Andrew S., Konstantinos Z.* Data assimilation: A mathematical introduction // Springer. 2015. 242 p.
6. *Fletcher S.J.* Data assimilation for the geosciences: From theory to application // Elsevier. 2017. 908 p.
7. *Leith C.E.* Numerical models of weather and climate // Plasma physics and controlled fusion. 1993. vol. 35. 919 p.
8. *Bellman R.E., Dreyfus S.E.* Applied dynamic programming // Princeton University Press. 2015. vol. 2050. 392 p.
9. *Pontryagin L.S., Boltyanskii V.G., Gamkrelidze R.V., Mishchenko E.F.* The mathematical theory of optimal processes // WileyEnglish. 1962. 360 p.
10. *Lions J.L.* Control optimal des systemes gouvernes par des equations aux derivees Partielles // Dunod. 1968. 426 p.
11. *Marchuk G.I.* Numerical methods in weather prediction // Academic Press. 1974. 288 p.
12. *Sasaki Y.* Some basic formalism in numerical weather analysis // University of Oklahoma. 1970. vol. 98. pp. 875–883.
13. *Penenko V.V., Obraztsov N.N.* Variational method of adapting of meteorological fields // Meteorology and Hydrology. 1976. vol. 11. pp. 3–16.
14. *Le Dimet F.-X., Talagrand O.* Variational algorithms for analysis and assimilation of meteorological observations: theoretical aspects // Tellus A. 1986. vol. 38. no. 2. pp. 97–110.
15. *Courtier P., Talagrand O.* Variational assimilation of meteorological observations with the adjoint equations. Part 2: Numerical results // Quarterly Journal of the Royal Meteorological Society. 1987. vol. 113. no. 478. pp. 1329–1347.
16. *Puri K. et al.* Implementation of the initial ACCESS numerical weather prediction system // Australian Meteorological and Oceanographic Journal. 2013. vol. 63. pp. 265–284.
17. *Rawlins F. et al.* The Met Office global four-dimensional variational data assimilation scheme // Quarterly Journal of the Royal Meteorological Society. 2007. vol. 133. no. 623. pp. 347–362.
18. *Courtier P., Thepaut J.-N., Hollingsworth A.* A strategy for operational implementation of 4D-Var, using an incremental approach // Quarterly Journal of the Royal Meteorological Society. 1994. vol. 120. no. 519. pp. 1367–1387.
19. *Marchuk G.I.* Adjoint equations and analysis of complex systems // Kluver Academic Publishers. 1995. 468 p.
20. *Marchuk G.I., Agoshkov V.I., Shutyaev V.P.* Adjoint equations and perturbation theory // CRC Press. 1996. 288 p.

21. *Kelly G., Thépaut J.-N.* Evaluation of the impact of the space component of the Global Observing System through Observing System Experiments // ECMWF Newsletter. 2017. vol. 113. pp. 16–28.
22. *Baker N., Daley R.* Observation and background adjoint sensitivity in the adaptive observation targeting problem // Quarterly Journal of the Royal Meteorological Society. 2000. vol. 126. pp. 1434–1454.
23. *Langland R.H., Baker N.L.* Estimation of observation impact using the NRL atmospheric variational data assimilation adjoint system // Tellus A: Dynamic Meteorology and Oceanography. 2004. vol. 56. no. 3. pp. 189–201.
24. *Lorenc A.C., Marriott R.T.* Forecast sensitivity to observations in the Met Office Global numerical weather prediction system // Quarterly Journal of the Royal Meteorological Society. 2014. vol. 140. no. 678. pp. 209–224.
25. *Soldatenko S., Tingwell C., Steinle P., Kelly-Gerreyn B.A.* Assessing the impact of surface and upper-air observations on the forecast skill of the ACCESS numerical weather prediction model over Australia // Atmosphere. 2018. vol. 9. no. 1. 23 p.
26. *Errico M.R.* Interpretations of an adjoint-derived observational impact measure // Tellus A: Dynamic Meteorology and Oceanography. 2007. vol. 59. no. 2. pp. 273–276.
27. *Lorenz E.N.* Deterministic nonperiodic flow // Journal of the Atmospheric Sciences. 1963. vol. 20. pp. 130–141.
28. *Lorenz E.N.* Irregularity: A fundamental property of the atmosphere // Tellus A. 1984. vol. 36. no. 2. pp. 98–110.
29. *Lupu C., Cardinali C., McNally A.P.* Adjoint-based forecast sensitivity applied to observation error variances turning // Quarterly Journal of the Royal Meteorological Society. 2015. vol. 141. no. 693. pp. 3157–3165.
30. *Cardinali C., Healy S.* Impact of GPS radio occultation measurements in the ECMWF system using adjoint-based diagnostics // Quarterly Journal of the Royal Meteorological Society. 2014. vol. 140. no. 684. pp. 2315–2320.
31. *English S. et al.* Impact of satellite data // European Centre for Medium-Range Weather Forecasts. 2013. vol. 711. 48 p.
32. *Jung B.-J., Kim H.M.* Adjoint-derived observation impact using WRF in the Western North Pacific // Monthly Weather Review. 2013. vol. 141. no. 11. pp. 4080–4097.
33. *Hoover B.T., Langland R.H.* Forecast and observation-impact experiments in the Navy Global Environmental Model with assimilation of ECWMF Analysis Data in the global domain // Journal of the Meteorological Society of Japan. 2017. vol. 95. no. 6. pp. 369–389.
34. *Cioaca A., Sandu A., de Sturler E.* Efficient method for computing observation impact in 4D-Var data assimilation // Computational Geosciences. 2013. vol. 17. no. 6. pp. 975–990.
35. *Janiskoca M., Cardinali C.* On the impact of the diabatic component in the forecast sensitivity observation impact diagnostics // Data Assimilation for Atmospheric, Oceanic and Hydrologic Applications. 2017. vol. 3. pp. 483–511.
36. *Sequeira L., Kirtman B.* Predictability of a low-order interactive ensemble // Nonlinear Processes in Geophysics. 2012. vol. 19. no. 2. pp. 273–282.
37. *Soldatenko S., Steinle P., Tingwell C., Chichkine D.* Some aspects of sensitivity analysis in variational data assimilation for coupled dynamical systems // Advances in Meteorology. 2015. vol. 2015. 22 p.
38. *Wittenberg A.T., Anderson J.L.* Dynamical implications of prescribing part of a coupled system: results from a low-order model // Nonlinear Processes in Geophysics. 1998. vol. 5. no. 3. pp. 167–179.