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ON THE EXPEDIENCY AND POSSIBILITIES OF APPROXIMATING A PURE DELAY LINK

Zhmud V., Dimitrov L., Sablina G., Roth H., Nosek J., Hardt W. **On the Expediency and Possibilities of Approximating a Pure Delay Link.**

Abstract. When solving problems of controlling an object with delay, it is often necessary to approximate a pure delay link with a minimum phase link in order to ensure the possibility of using analytical methods for regulator design. There are many approximation methods based on the Taylor series expansion, as well as modified methods. The most famous one is the Padé approximation method. The known approximation methods have significant drawbacks, which this paper reveals. However, there are other methods of forming other types of filters that can serve as a better approximation in determining the delay relationship, although they are not used for these purposes. In particular, methods of forming the desired differential equation of a locked-loop system of a given order by the method of numerical optimization are known. In this case, the locked-loop system behaves like a filter of the corresponding order, the numerator of which is equal to one, and the specified polynomial is in the denominator. Modeling has shown that such a filter is an effective alternative approximation of the delay link and can be used for the same purposes for which it was supposed to use the Padé approximation. The polynomial coefficients in the literature were calculated only up to the 12th order. The higher the polynomial order is, the more accurate the approximation is.

Keywords: Padé formula, delay, approximation, control, automation.

1. Introduction. The development of numerical methods makes it possible to easily and efficiently design a controller for a locked-loop system if the mathematical model of the object is known with sufficient accuracy (as a rule, it is sufficient to know the basic parameters of the model with an error of at least 1%). However, many research teams are actively developing analytical methods for the synthesis of controllers, including for objects containing a pure delay link in their model. This does not seem to be extremely relevant since such software tools as MATLAB, Simulink, MathCAD, SimInTech, VisSim allow simulation of a pure delay link without any approximations. Nevertheless, papers on such an approximation are published [1–7], dissertations are defended [8–10], and this direction is widely lobbied by various research teams [11–17]. Considerations against using such an approximation are as follows: a) a pure delay link is characterized by a linear dependence of the delay on frequency, any approximation by a filter cannot provide such a dependence; b) well-known software tools and methods for designing regulators based on their use do not require any approximation, as they easily simulate the delay link inaccuracy; c) each approximation introduces an error in the calculations and the results of these calculations. Therefore, the design result with such an approximation may contain such a significant error that it will be inapplicable.

However, it would be wrong to reject any research carried out using such an approximation since a simple simulation experiment shows that in a certain area of problems this approximation allows obtaining the required result in a fairly simple way. In this case, it is necessary not to reject any such approximation completely, but it would be useful to determine the limits of its applicability for solving problems of controller design, and this is the purpose of this paper. The approximation of a pure delay link by a transfer function in the form of a rational fraction is still widely used to implement the possibilities of designing controllers for objects with delay [1–17]. Almost all real objects have a delay, and analytical design methods are still used by many researchers (despite the rapid development of numerical methods [18–22]), the problem of such an approximation is of interest at least from the standpoint of an adequate assessment of the results, usefulness and scientific contribution of dissertations and new publications using such an approximation. The Padé approximation is often called the most adequate in the literature, arguing that it best meets the design objectives of the controller [11–17]. This statement is justified for objects with large inertia that is not associated with this delay. But this is not always the case. There are object models for which such an approximation cannot be used. For example, if the object model is in a series connection of an integrator and a delay link, then the Padé approximation is not efficient enough. This could be verified by numerical modeling. In the case of application of the method of numerical optimization, such an approximation does not allow solving the problem of regulator design for such objects.

2. Task Statement. Let us consider a control object whose mathematical model has a dependence of the output value y(t) on the input action u(t), known in the form of the transfer function W(s), which in addition to the minimum phase part also contains a link of pure delay e-τs. Here s is the argument of the Laplace transform. The object is affected by an unknown disturbance h(t), which causes a change in the output value Δy(t). The effect of this disturbance can be described as an uncontrolled addition to the output value, which gives the output a modified value of z(t). It is necessary to design a control system for this object so that the output value repeats the prescription v(t) as accurately as possible and, as far as possible, does not depend on interference the h(t). This is the classical formulation of the automatic control problem, which is most successfully solved by creating a loop with a negative feedback unit with a controller in a front loop, as shown in Figure 1. As a rule, the problem of design involves finding a mathematical model of the controller. If the controller is sought in the form of a PID controller, i.e. a structure containing proportional, integrating and derivation links connected in parallel so that the output signals are summed and fed to the object, then the design problem comes down to finding the coefficients of these links. As a rule, the controller should provide the highest achievable speed, zero static error and, if possible, the minimum overshoot, no more than 10%.

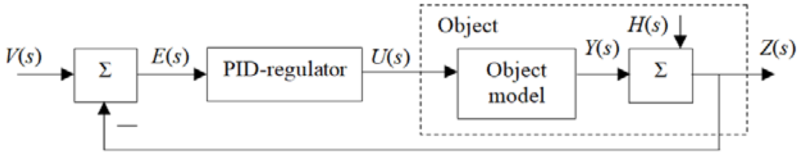

Fig. 1. A typical automatic control system with a sequential PID controller in a direct loop: values are given in terms of Laplace transforms of time functions

Among the methods for solving this problem can be distinguished analytical and numerical methods. Analytical methods deal with differential equations. The presence of a delay link complicates the application of such methods, as does the presence of nonlinearities and other problematic elements. Numerical modeling methods are effective for any of the most complex models of an object. In this case, the presence of any nonlinearities delays or other features of the object model is not in the list of problems, as all these elements are easily simulated by any of the above software.

The paper [21] provides information on two types of approximation. The first one is based on the expansion of the transfer function of the pure delay link in the Taylor series. The second one is the Padé approximation which differs in the presence in the numerator of the transfer function of a polynomial of the same order as in the denominator, but the terms with an odd power of the argument, Laplace transforms in this model have a negative sign. Paper [21] proposed its own approximation, which is said to be free from the drawbacks of the two indicated models. Throughout the entire cycle of works, including [1–22], the approximation is used to calculate the controller for the object with transfer function containing the following multiplier:

$$W_D(s) = e^{-\tau s}. \quad (1)$$

Here τ is the time constant. Various models of the transfer function have been proposed, and according to many authors, this one is the closest one to the transfer function of this link. The transfer function (1) is generally substituted by a rational fraction in the form of the ratio of two polynomials in the argument s.

$$W_M(s) = \frac{B(s)}{D(s)} = \frac{\sum_{i=0}^{m} b_i s^i}{\sum_{j=0}^{l} a_j s^j}. \quad (2)$$

Here $B(s)$ and $D(s)$ are polynomials in s. The degrees of these polynomials are equal to m and l respectively. For the transfer function (1) to be physically realizable, the order of the numerator must be less than the order of the denominator: l > m. However, for controllers and other elements that have much faster performance than other elements in the same control system, it is allowed that the order of the numerator is equal to the order of the denominator: $l = m$. In some cases, transfer functions are used in which the order of the denominator is one unit larger than the order of the numerator. For example, in an ideal derivative link: $l=m+1$.

Taylor's method is based on power series. For the transfer function of the delay link (1), the Taylor series expansion for a function of the form (1) gives:

$$W_T(s) = \frac{1}{1+\tau s+\frac{1}{2}(\tau s)^2+\frac{1}{6}(\tau s)^3+\cdots}. \quad (3)$$

Many publications, including [12, 14, and 21], preference is given to the Padé approximation, which has the following form:

$$W_\tau^N(s) = \frac{\sum_{k=0}^{N} \frac{(n+k)!}{k!(n-k)!}(-s\tau)^{n-k}}{\sum_{k=0}^{N} \frac{(n+k)!}{k!(n-k)!}(s\tau)^{n-k}}\,. \quad (4)$$

Here N is the order of approximation of the model, n is the order of the polynomial in the denominator of the model, in this case, N = n.

The exact value of the second and subsequent coefficients in (4) depends on the order of approximation. The reason for this preference is not always clear. In [21] it is mentioned that the possibilities of the second option are greater, as it has a larger number of variable coefficients. We can’t agree with this because the number of varying coefficients in relations (3) and (4) for the same order of the denominator exactly coincides, because in relation (4) the coefficients in the numerator in absolute value coincide with the coefficients of the denominator, and their sign alternates, starting with a positive one. If, in this case, the denominator of relation (4) is specified, then its numerator is also completely finally specified and no additional variation of the coefficients is possible. Paper [21] suggests the model in the following form:

$$W_{VIM}(s) = \frac{b_0+b_1 s}{1+a_1 s+a_2 s^2}. \quad (5)$$

The unexpected fact is that in model (5) the free term of the numerator is given by a letter and it is determined in the denominator by a specific value equal to one. The fact is that the ratio of free terms determines the steady-state value of the response, and for relation (1) it is strictly equal to one. Therefore, the free term in the numerator must also be equal to one, and it does not need to be calculated by any method, b_0=1. It is obvious that for the case τ=1 the paper [21] has obtained a relation where this is fulfilled:

$$W_{VIM}(s) = \frac{1-0.317s}{1+0.683s+0.184s^2} \quad (6)$$

Figure 2 shows the transient processes when a unit step is applied to elements that have transfer functions (1), (3), (4) and (6) for $N = 4$. The response of the link with the transfer function (1) is ideal. The response according to relation (3) is excessively stretched in time. The response according to relation (4) has a very large starting value (which cannot take place in the original model) and a very large reverse overshoot. In this case, the response of link (6) seems to be most closely similar to the response of an ideal link (1). However, it is far from the ideal case, it stands at an intermediate position between the responses of elements (3) and (4), combining their disadvantages and advantages equally. The undoubted advantage of this model is the fact that at the zero moment the response is zero, which is more consistent with the true model (1).

This task aims to address the following issues:

A. How appropriate is the use of approximation?

B. Is not it better to abandon the use of methods that cannot do it without approximation, and to use only those methods that use an exact delay model?

C. If the proposed approximation methods can still be used, then under what conditions?

D. If it is appropriate to use the approximation, is it possible to improve it to expand the scope of its applicability, and if so, how exactly?

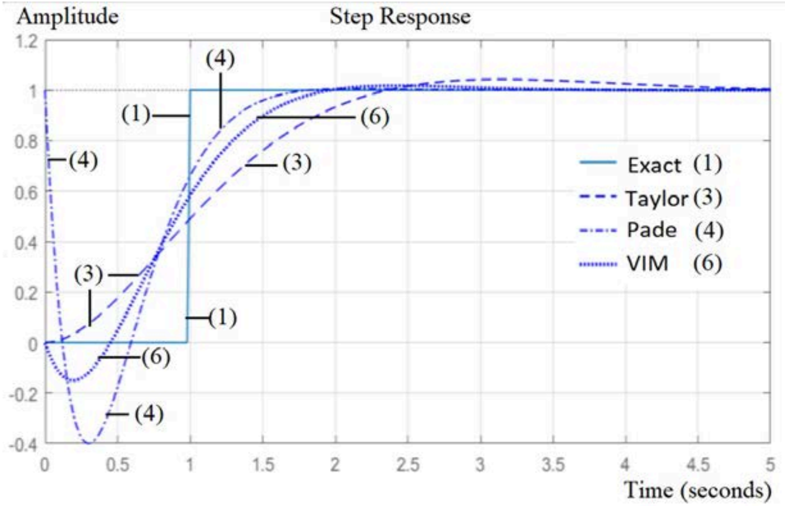

Fig. 2. Comparison of responses of models (1), (3), (4) and (6) from publication [21], order of models $N = 2$, the designation number is the same as the equation number

3. The Proposed Research Method. It is proposed to solve the problem by a method of numerical optimization. The graphical programming structure for solving this problem is shown in Figure 3. It contains a system model, a setting signal generator, a cost function estimator, an optimizer, and an oscilloscope. In this structure, an exact model of the object is used, and no approximations are required. To address the question of how competent the delay approximation is when using analytical methods, it is proposed to use this approximation when using the method of numerical optimization and then simulate the system with the obtained controller and use the exact model of the object. If the result of such a simulation is sufficiently similar to the result of the calculation by the model, it can be argued that such an approximation can be used. But if the result obtained by the numerical optimization method is still better than the result obtained using the approximation, we can supplement this conclusion with the statement that although the approximation can be used, it still does not give an optimal solution; thus, it is inappropriate.

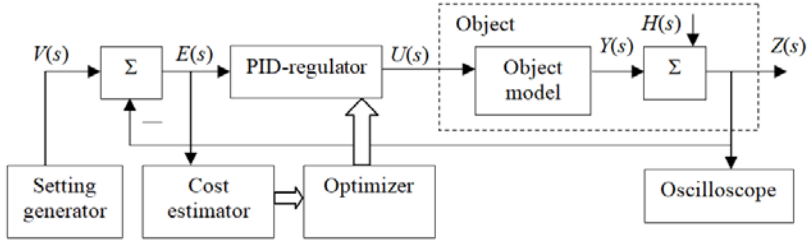

Fig. 3. The structure of graphical programming for solving the problem of numerical optimization of the controller

For modeling, the VisSim software was chosen for the reason that the calculation method in it coincides with the method of operation of any digital controller. Namely, the simulation is carried out in steps. This simplest advantage turns out to be decisive in comparison, for example, with the use of the MATLAB software, which allows performing analytical calculations in the form of finding the necessary functions, but at the same time, it does not form those delays that occur in a real system due to the real operation of a digital controller, and it does not form those errors in the estimation of the derivative and integral of the function by its readings, which also occur in a real system with a digital controller. Many problems can be specified that the MATLAB program solves efficiently and accurately, but the result obtained is not applicable in practice since such accuracy is not achieved in a real system. With the use of the VisSim software, this does not happen: if the software gives a result, the same result will take place in practice since in a real system the same step-by-step algorithms for calculating errors and control signals will be used.

4. Results of Studies. Comparison of the type of transient processes, which are the model's response to a single stepwise jump, shows that the models (3), (4) and (6) are very far from ideal. Indeed, the response of the Taylor model (3) is unnecessarily prolonged in time (see Figure 1), the response of the Padé model (4) is characterized by two drawbacks: firstly, a nonzero value at the moment $t = 0$, equal to the amplitude of the input signal, and secondly, significant reverse overshoot, reaching 40% of this amplitude. The response of the model (6) seems to be the best at first glance, as it starts from zero, the reverse overshoot is less than in the response of the model (4) and amounts to 15% of the input signal amplitude. In terms of rising time, this response occupies an intermediate position between the responses of the models (3) and (4). However, this characteristic is insufficient for making a decision that the model (6) is the best. Filters are described in [22]:

$$W_{CH}(s) = \frac{1}{D_{CH}(s)}. \quad (7)$$

The denominator of the fraction (7) contains polynomials. The lowest and highest coefficients are strictly equal to one. The remaining coefficients are found by numerical optimization using the following objective function:

$$F(T) = \int_0^T \left\{k_w f\left[e(t)\frac{de}{dt}\right] + |e(t)|t\right\}dt. \quad (8)$$

Here k_w is a weighting factor, $e(t)$ is the difference of the filter output signal from unity, f is a limiter function that excludes negative values, which can be specified by the expression:

$$f[x] = \max\{x, 0\}. \quad (9)$$

In [22], this coefficient is set equal to one and $k_w = 1$. These polynomials are called Chegorsky polynomials. However, the detailed studies have shown that it is very expedient to take a much larger value of this coefficient, for example, $k_w = 100$.

The coefficients of the polynomial $D_{CH}(s)$ can (and it is recommended) be found even if it is given in the simulation by the product of several polynomials [22]. For example, several polynomials of the third order, for the case when the order of the polynomial is a multiple of three. If the order of the polynomial is not a multiple of three, then some of the polynomials that are factors may have a lower order, for example, the first or the second. The paper [22] gives the results of calculating the coefficients of the polynomials $D_{CH}(s)$ up to order 12 inclusive. Although these coefficients are not given in general terms, they are only represented by numerical values. This result can be fully used in modeling or analytical calculations. In addition, in [22], the polynomials $D_{CH}(s)$ are presented as products of polynomials of lower order. This is also not a problem because, if necessary, all coefficients can be calculated by simple multiplying elementary polynomials. In addition, finding the roots of polynomials in this form is much easier, and this is irrelevant in modeling, as it is possible to use any variant of writing polynomials. It is easy to use the polynomial of the highest order among all $D_{CH}(s)$ polynomials published so far. Its analytical expression is as follows:

$$P_{CH12}(s) = (1 + a_1 s + a_2 s^2 + s^3)(1 + a_3 s + a_4 s^2 + s^3)(1 + a_5 s + a_6 s^2 + s^3)(1 + a_7 s + a_8 s^2 + s^3). \quad (10)$$

In this case, the values of the coefficients are as follows: a_1=2.02952, a_2=4.36457, a_3=5.51585, a_4=5.1134, a_5=4.47914, a_6=10.5786, a_7=1.19969, a_8=3.07125 [22]. It is noticed during modeling that the response of the filter (7) with polynomial (10) in the denominator models the response of a pure delay link with high accuracy. To demonstrate this, it is sufficient to use the *VisSim* software with the structure of the model shown in Figure 4. The simulation results are shown in Figure 5. It shows that this graph perfectly approximates the response of the delay link with a time constant equal to τ=14 *s*.

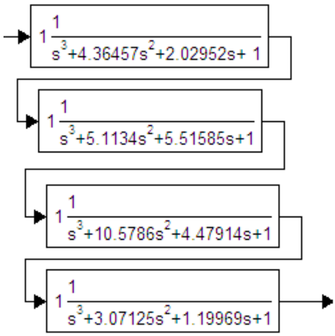

Fig. 4. Structure for modeling filter (7) with polynomial (10)

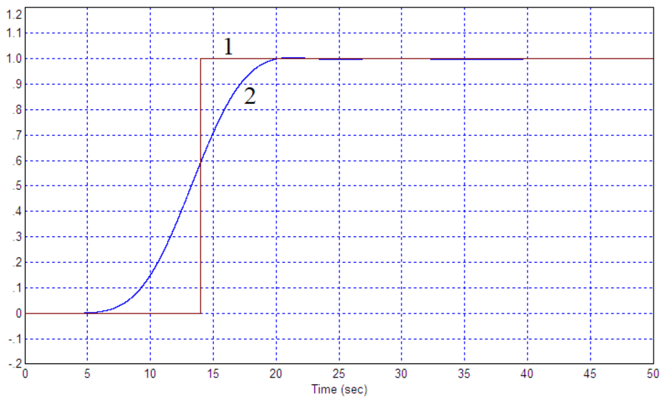

Fig. 5. Transient process in the structure according to fig. 2 (line 2) versus the ideal process (line 1)

These two graphs coincide in the initial interval from 0 to 5 *s*, in the final interval from 20 *s* and further to infinity, as well as at the point t=14 *s*. The value of the time constant τ = 14 *s* is just found from the condition that at the moment of the jump the indicated function reaches 60% of its steady-state value, which takes place in Padé approximations of any order. Thus, filter (7), (10) can be used as a model for the delay link (1) at τ=14 *s*. For the case of an arbitrary value of τ, it is sufficient to apply the appropriate scaling of frequencies and time. For example, if we assume that the unit of measurement on the graph in Figure 5 is the time interval equal to 14 *s*, then this graph corresponds to the model of the delay link with τ=1 (in new units of time). Note that even the seventh-order polynomial $D_{CH}(s)$ in all parameters better approximates the delay link than approximation (6). For the case τ=1, this polynomial has the following form:

$$P_{CH5}(s) = (1 + 0.4123s + 0.11493s^2 + 0.00463s^3)(1 + 0.5113s + 0.02778s^2). \quad (11)$$

Also, for comparison, the 9th order polynomial was simulated for the case τ=1. This polynomial has the form:

$$P_{CH9}(s) = (1 + 0.113s + 0.13s^2 + 0.001s^3)(1 + 0.4321s + 0.02915s^2 + 0.001s^3)(1 + 0.3824s + 0.08446s^2 + 0.001s^3). \quad (12)$$

Figure 6 shows the structures for modeling the corresponding filters, and Figure 7 shows the resulting transients. It is obviously from Figure 7 that model (6) is the worst approximation of model (1) in comparison with models (11) and (12).

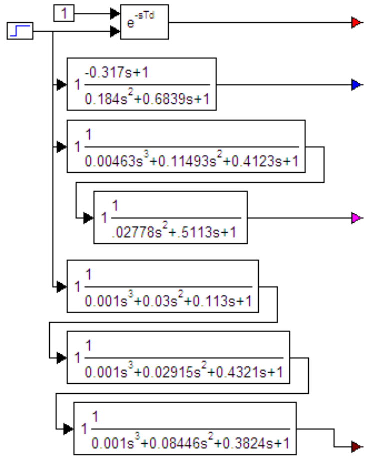

Fig. 6. Structures for modeling filters (1), (6), (11) and (12)

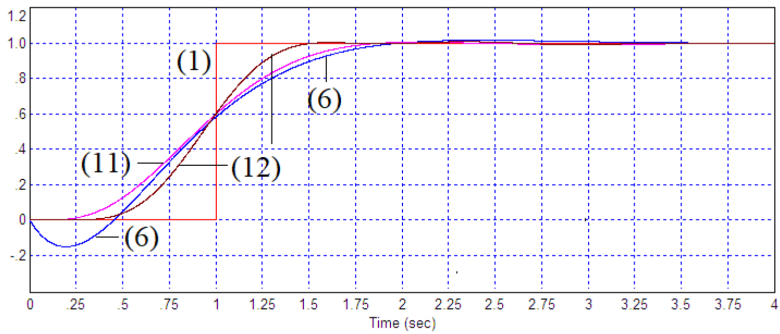

Fig. 7. Transient structures in the models according to relations (1), (6), (11) and (12), the marking of the lines corresponds to the number of the model equation

5. Control of the Object in the Form of an Integrator and a Delay Link. Using the known models (4) and (6), as well as the proposed model (10), we calculate the controller for the object with delay, after which we will simulate the system containing such a controller and the actual model of the object according to relation (1). In the same way, let us check the applicability of the Padé model and other models.

Consider an object of the form

$$W_{O1}(s) = W_M(s)W_D(s) = \frac{1}{s}e^{-\tau s}. \quad (13)$$

Let pose the problem of finding the PID controller coefficients for controlling this object in a locked loop. The regulator transfer function is:

$$W_R(s) = k_P + k_I \frac{1}{s} + k_D s. \tag{14}$$

The regulator coefficients k_P, k_I, k_D have to be calculated. We will use the numerical optimization method.

For optimization, the next experiment uses the objective (cost) function (8), (9). Initial parameter values can be set to zero. Since model (10) with the above coefficients best approximates the delay link at τ=14 *s*, this value of τ is put in relation (10). This is acceptable since this result can always be recalculated for a different time constant by introducing a scale factor. With the new coefficient, the overall gain of the object will change. This is also insignificant since the change in the coefficient can be easily taken into account by proportional changes in all controller coefficients; therefore, problem (10) remains the most general for any arbitrary finite value of τ. The structure for designing a regulator for this object is shown in Figure 8. It fully corresponds to the structure of Figure 3 in terms of graphical programming in the VisSim software. The upper part of this figure shows the structure of the control system that will be in the implementation of this system in reality. It has a generator for the reference signal (in this case, it is a step jump), an adder through which the negative feedback is closed, a PID controller and an object. In this case, instead of an object, there is its model, which consists of a model of a delay link and an integrator. The middle and lower parts of Figure 8 show the blocks for calculating the coefficients of the controller and the calculator of the value of the cost function. As a result of the optimization, the following values of the PID controller coefficients were obtained: k_P=0.0572943, k_I=-3.48581·10^{-7}, k_D=0.322082. These factors can be rounded up to 3–4 significant digits. The coefficient of the integrating path is very small, and this is natural since the object contains an integrator, therefore, the integrator is not needed in the controller. However, for the sake of the purity of the experiment, the calculated coefficients are saved without rounding. The transient process obtained in this system is shown in Figure 9.

Now the approximation (10) in the structure shown in Figure 8 is replaced by the exact model of the object (11). The obtained transient process in the new system is shown in Figure 9 (line 2), where the process obtained by the approximation is shown for comparison (line 1). It can be seen that the process in the system with a real object containing a link of a pure delay differs from the process obtained in the system with its approximation, but this difference is not significant since the overshoot does not exceed 10%.

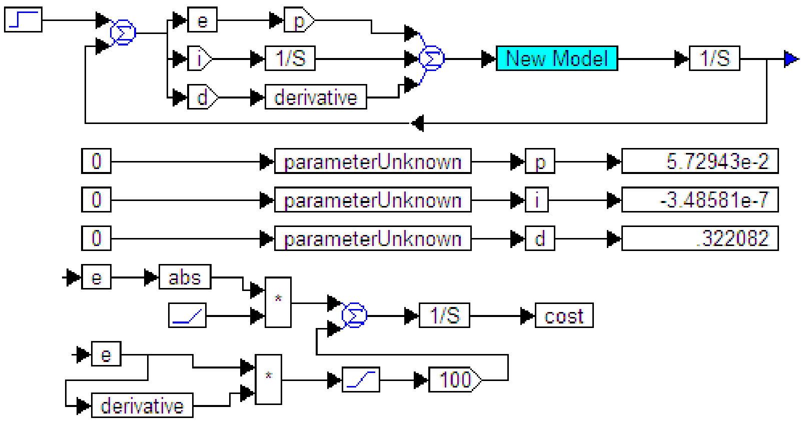

Fig. 8. Structure for designing a regulator for an object (11)

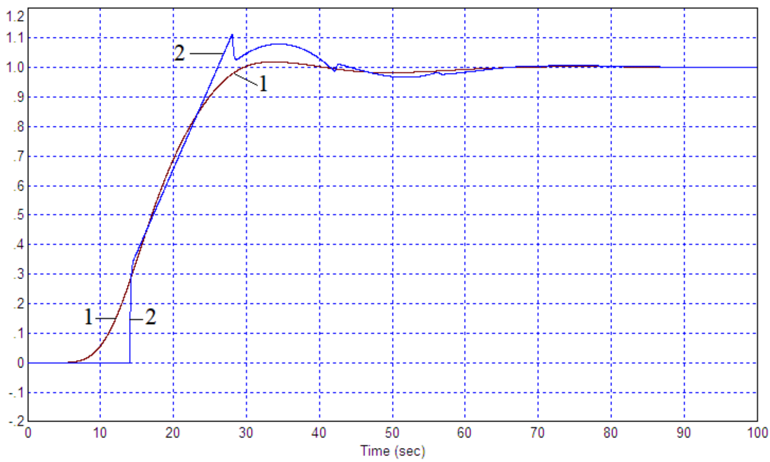

Fig. 9. Comparison of the transient process obtained using the approximation (10) (line 1) and using the exact description of the object (11) (line 2)

The comparison of the two transient processes shows that the process with the actual delay begins only after this delay time has elapsed, which is quite natural. Meanwhile, the process with approximation begins earlier, because the response in this approximate model also begins earlier, as shown in Figure 5. The response in a real system contains several small jumps, but in general, the response is quite similar to the response in a system with an approximation. The largest difference is no more than 10%, except for the initial section, where the difference is inevitable but also not critical, as the type of the transition process in this initial section should be different in this way as well. If the Padé approximation (4) is applied in the considered and detailed method, then, in this case, the optimization with the help of this objective function will not lead to a result. The VisSim program cannot complete the calculations because the calculation results in unacceptably large values of the output signals and the objective function. The use of approximation (6) in solving this problem by the considered method also does not lead to success.

The above shows that the advantage of the proposed approximation in the form of a filter based on the polynomial $D_{CH}(s)$ model has a wider range of applicability. In particular, if the plant model consists of a delay link and an integrator, then the Padé approximation does not allow finding the PID controller by numerical methods. Some analytical methods may not give the desired result. If, in this case, a filter (7) is used as a model, then the required optimization result is achieved. In this case, the optimization procedure allows finding the optimal settings for the PID controller, with which the real system also works successfully.

Note that the use of the exact model (13) in the structure shown in Figure 8 easily leads to finding the required optimal regulator.

6. Control of the Object in the Form of an Integrator and Aperiodic Link. Consider an object of the form:

$$W_{O2}(s) = \frac{1}{s+10} e^{-\tau s}. \tag{15}$$

Let's solve the same problem in the same two ways. The experiment used the fifth-order Padé approximation for τ=14 *s*:

$$W_{1\tau}^{5}(s) = \frac{-s^5+30s^4-420s^3+3360s^2-15120s+30240}{s^5+30s^4+420s^3+3360s^2+15120s+30240}. \tag{16}$$

Figure 10 shows the results for calculating the optimal controller using approximation (14) (line 1) and the results of using this controller with a real object (11) (line 2). If we neglect the initial part, which must be different, then the rest of the transient processes coincide almost perfectly. The maximum difference between the actual process and the process calculated with the Padé approximation is 5%, which is a very good result because overshoot is three times less than required, it does not exceed 3%. The largest jump occurs by 5% and does not contribute to overshoot, the difference in the process at the initial stage, when the delay is in effect, is inevitable, and it is exactly what it can be, that is, before the expiration of the delay time, there is no signal at the output, which is natural. When using the filter approximation (10), the results are approximately the same as in the case of the model with an integrator, i.e. it almost coincides with the processes shown in Figure 7. In this case, the maximum difference also reaches 10%, as for the object (10).

Consequently, the proposed model (10) has both disadvantages and advantages. The advantages have been mentioned above. The disadvantage of the model is that when using it, the obtained result coincides with less accuracy with the result obtained using the Padé model.

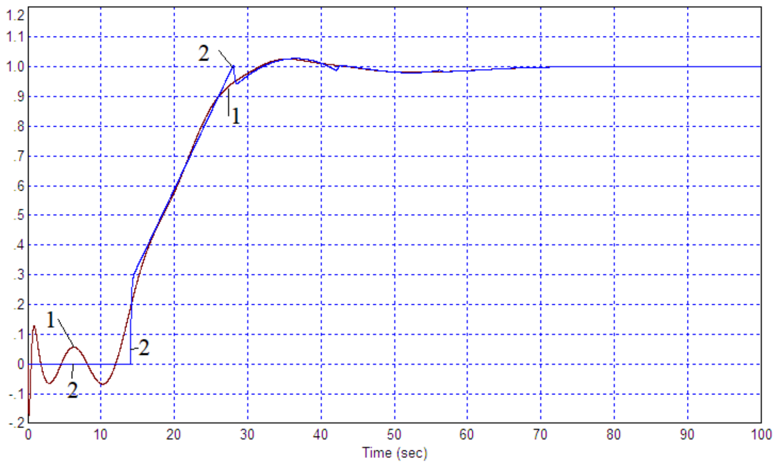

Fig. 10. Comparison of the transient process obtained using the fifth-order Padé approximation (line 1) and using the exact description of the object (11) (line 2)

7. The Use of Mixed Approximation. It was found that the serial connection of the Padé approximation model and the filter model (10) gives a much more satisfactory result. The transfer function of the composite approximation WC (s) in this case takes the following form:

$$W_C(s) = W_{CH12}(s)W_{1\tau}^{5}(s). \quad (17)$$

The transient process at the output of the synthetic model (17) is shown in Figure 11. It corresponds to a time delay of τ=28 *s*.

An even better result is achieved when using the $W_{C2}(s)$ approximation in the form of a series connection of two models: the Padé approximation and filter (10) one:

$$W_{C2}(s) = W_{1\tau}^{5}(s) W_{CH12}(s) W_{1\tau}^{5}(s). \quad (18)$$

It is possible to successfully solve the object control problem of the form (13) when set τ=42 *s*. Figure 12 shows the structure for modeling and optimization of a system with an object in form (13) using the compound approximation (18).

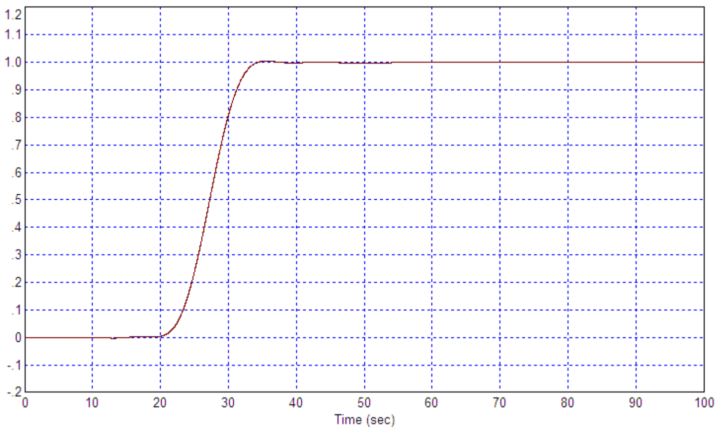

Fig. 11. Transient process at the output of the synthetic model

Figure 13 shows the transient processes calculated in the system with object (13) where approximation (16) is used instead of a pure delay (1) (line 1) and when the system has a real object (13) (line 2). Comparison of these graphs shows that the coincidence of the two processes is almost ideal with the exception of small inevitable deviations near the points t_1=τ, t_2=2τ, t_3=3τ.

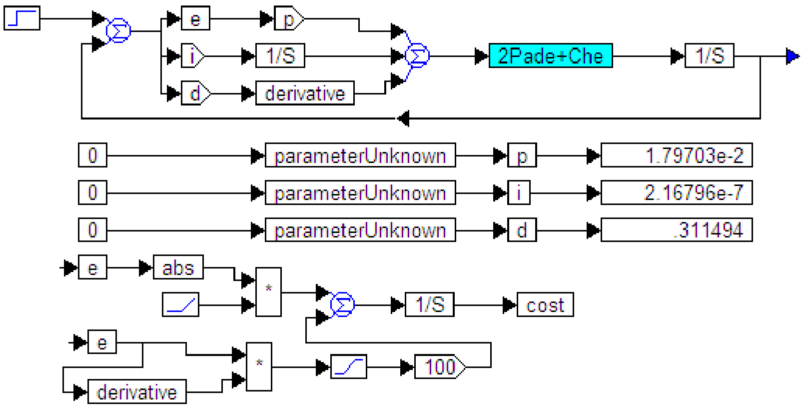

Fig. 12. Structure for modeling and optimization of a system with an object of the form (13) using compound approximation (18)

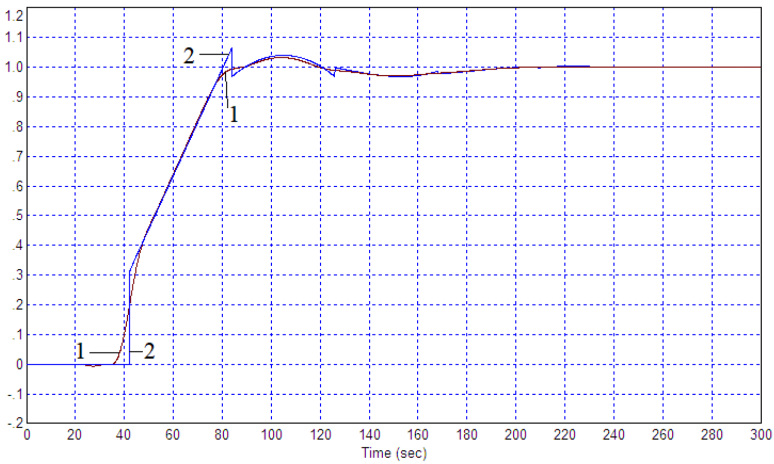

Fig. 13. Transient processes calculated in the system with approximation (18) (line 1) and in the system with a real object (10) (line 2)

Discussion. Figure 7 shows that the proposed approximation by the filter based on the polynomial (10) is better because the solution of the control problem for an object containing an integrator and a delay link can be successfully solved only by using this approximation. The rest of the known approximations, including the Padé approximation, do not work in this case when using the numerical optimization method. It follows that even when analytical methods are used, these approximations may not be applicable or work worse than the approximation based on the polynomial (10).

Composite approximation (18) is even more accurate. The use of the two proposed approximations (10) and (18) allows for a more successful solution of the tasks for designing controllers by the methods of numerical optimization. In this case, the approximation (10) is not without drawbacks since there are models of objects, such as model (13), for which the use of the Padé approximation (4) gives more accurate results. But the advantage of approximation (10) is that there are such object models as a model according to relation (13), for which the Padé approximation does not allow solving the posed problem of regulator design, while the approximation (10) allows solving this problem effectively. Also, a composite approximation (17) and (18) is proposed, and it is much more accurate, and it allows solving all the problems considered with the highest accuracy. The accuracy of the solution of the problem was evaluated by the accuracy of the coincidence of transient processes in the resulting system using an approximation model and similar processes with the same controller, using the exact model of the object. In the case of using model (15), this accuracy is the highest one.

The proposed approximation can also be compared with other options, for example, with a filter approximation with a binomial polynomial in the denominator, or an approximation based on the Taylor series, as (3), used in publication [24], or with a polynomial of the following form [25]:

$$G(s) = \frac{1}{1+s+0.5s^2}. \tag{19}$$

All of these approximations are much less accurate and, thus, perform worse. The comparison of the integrals of the modulus of the error of these approximations with the same characteristic for the approximation based on the polynomial (10) gives the error that is 1.6 times greater.

Conclusion. The paper proposes a new approximation of the delay link with a low-pass filter. The approximation of the transcendental transfer function by a link in the form of a rational fraction of polynomials makes more efficient the use of analytical methods for designing a controller for objects with delay. The disadvantages of the Padé approximation are shown by the method of modeling and numerical optimization and three new types of approximation, free from the identified shortcomings, are proposed. Based on the research, it can be said that the known methods for approximating the delay link by a polynomial transfer function have their drawbacks. The expediency of the most accurate approximation can take place only if it is necessary to use analytical methods of design; since numerical methods do not need such an approximation, all known packages for modeling and optimization easily simulate the exact transfer function (1). Thus, approximation by known models is far from always advisable, but only if analytical methods are used further, the known approximation methods are inferior to the proposed method.

Calculations of the proposed approximation up to the ninth order have been carried out. Increasing the order leads to a more accurate approximation of the stepwise delay jump. Figure 14 shows transient processes in filters of the fifth and sixth order, Figure 15 shows the processes in filters of the eighth and ninth order.

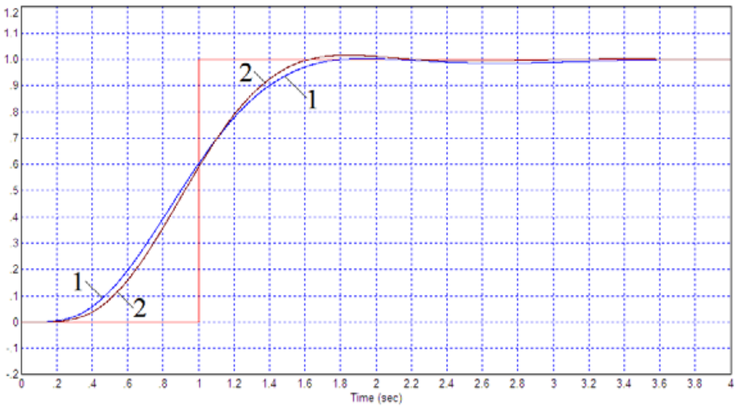

Fig. 14. Transient processes in filters of the fifth order (line 1) and sixth order (line 2) according to the proposed method

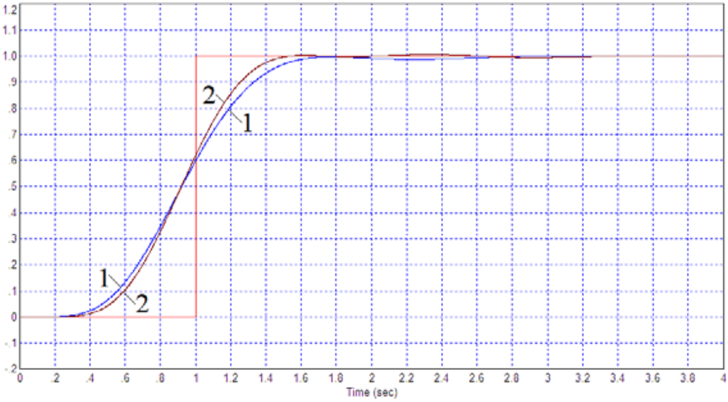

Fig. 15. Transient processes in filters of the 8th order (line 1) and 9th order (line 2) according to the proposed method

Raising the order above the fifth does not give a noticeable advantage in optimization, since fairly reliable results are obtained even when using models of the 3rd – 5th order. Therefore, such an excessive complication of the model by increasing the order of approximation is apparently inappropriate.

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О ЦЕЛЕСООБРАЗНОСТИ И ВОЗМОЖНОСТЯХ АППРОКСИМАЦИИ ЗВЕНА С ЧИСТЫМ ЗАПАЗДЫВАНИЕМ

Жмудь В.А., Димитров Л., Саблина Г.В., Рот Г., Носек Я., Хардт В. **О целесообразности и возможностях аппроксимации звена с чистым запаздыванием.**

Аннотация. При решении задач управления объектом с запаздыванием часто необходимо аппроксимировать звено чистого запаздывания минимально фазовым звеном, чтобы обеспечить возможность использования аналитических методов для проектирования регулятора. Существует множество методов аппроксимации, основанных на разложении в ряд Тейлора, а также модифицированных методов. Наиболее известен метод аппроксимации Паде. Известные методы аппроксимации имеют существенные недостатки, которые выявляет данная работа. Однако существуют и другие способы формирования других типов фильтров, которые могут служить лучшим приближением при определении соотношения задержек, хотя они и не используются для этих целей. В частности, известны способы формирования искомого дифференциального уравнения замкнутой системы заданного порядка методом численной оптимизации. В этом случае замкнутая система ведет себя как фильтр соответствующего порядка, числитель которого равен единице, а указанный полином стоит в знаменателе. Моделирование показало, что такой фильтр является эффективной альтернативной аппроксимацией звена задержки и может использоваться для тех же целей, для которых предполагалось использовать аппроксимацию Паде. Полиномиальные коэффициенты в литературе рассчитывались только до 12-го порядка. Чем выше порядок полинома, тем точнее аппроксимация.

Ключевые слова: формула Паде, запаздывание, аппроксимация, управление, автоматизация.

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