UDK 62.50 **DOI** 10.15622/ia.22.3.6

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MINIMIZATION OF PEAK EFFECT IN THE FREE MOTION OF LINEAR SYSTEMS WITH RESTRICTED CONTROL

Dudarenko N., Vunder N., Melnikov V., Zhilenkov A. **Minimization of Peak Effect in the Free Motion of Linear Systems with Restricted Control.**

Abstract. A peak effect minimization problem in the free motion of linear systems is considered in the paper. The paper proposes an iterative procedure for the peak effect minimization using a combination of the recently proposed gramian-based approach and the theory of using the condition number of an eigenvectors matrix for the upper bound estimations of the system state processes.

Minimization of control costs is based on the analysis of the singular value decomposition of a gramian of control costs, followed by the formation of major and minor estimations of the gramian. Minimization of peak effect in the trajectories of free movement of systems is carried out by minimizing the condition number of the eigenvectors matrix of the matrix of a stable closed-loop system, while the state matrix with the desired eigenvalues and eigenvectors is designed with the generalized modal control. The development of an iterative algorithm for the peak effect minimization in the trajectories of linear systems under any non-zero initial conditions with restricted control is based on an aggregated index. The index takes into account both the estimate of the gramian of control costs and the condition number of the eigenvectors matrix of the stable closed-loop system. Minimization of the aggregated index makes it possible to ensure minimal deviations in the trajectories of free movement of systems of the considered class.

The procedure is applied to a system of two satellites with restricted control, where peak effects in satellites relative trajectories are minimized. Two cases of peak affect minimization are considered. In the first case, the peak effect minimization in the trajectories of free movement of satellites is carried out only by minimizing the gramian of control costs. In the second case, the peak effect minimization is realized using the developed algorithm. The results illustrate the efficiency of the procedure and indicate the decrease of the peak effect for the satellites relative trajectories.

Keywords: condition number, control costs, restricted control, free motion, gramian, peak effect, satellites, upper bounds.

1. Introduction. In recent years, the peak effect problem is actively investigated [1 – 4]. Peak effects in the free motion of a linear system occur due to nonzero initial conditions in the absence of an exogenous input signal. The problem is not new. First of all, the relationship between system poles and the behaviour of the transition process of a system were investigated by A.A. Feldbaum in the paper [5], which initiated the research of the peak effect problem. Then, the problem of large deviations was continued in the works [6] and [7], where the relationship between the peak effect level and the transient attenuation rate was revealed. Later, the peak effect was also found in systems, where poles had a different location from the one that causes an increase in the attenuation rate of the transient process [8]. The problem was presented for switching systems in [9] and for cascade control systems in [8], where the result of R.N. Izmailov was generalized to obtain estimations of the deviations for the outputs. Recent papers [3, 10] continued the study in that field for different values of system poles, and new results for estimations of the upper bound of deviations were obtained with the linear matrix inequality [3] and with the condition number of the eigenvectors matrix [10]. Also, it was investigated that the peak effect depends on a matrix representation.

The peak effect minimization problem is also relevant to fluid flow control. The scientists James F. Whidborne and John McKernan consider an equivalent problem to provide minimization of the transient energy growth [11], that is used actively nowadays in the fluid flow control field [12 – 15].

The peak minimization problem is actual for the stabilization systems [3] and the tracking systems [9]. In this paper, the problem of peak effect minimization for stabilization systems with restricted control is considered. The peak effect in the researched system appears due to nonzero initial conditions and restricted control. Therefore, the aim of this paper is to propose a procedure for the peak effect minimization for stabilization systems with restricted control. The procedure is based on a combination of the recently proposed gramian-based approach and the theory of the usage of the condition number of an eigenvectors matrix for the upper bound estimations of system processes. The proposed procedure is applied to a satellite system, where the effectiveness of the procedure is illustrated.

The paper is laid out as follows. In Section 2, the approach for the estimation of peak effect in the free motion of linear continuous-time systems based on the calculation of the condition number of the eigenvectors matrix is described. Then, the gramian-based method for the estimation of control costs is discussed in Section 3. An iterative procedure for the minimization of the peak effect in the free motion of linear continuous-time systems with input saturation is proposed in Section 4. Then, the system of the two satellites' relative motion is described in Section 5 and the modelling of the system is presented without taking into account restricted control. In Section 6 the proposed procedure is applied to the satellite system with restricted control to provide the peak effect minimization. The results are discussed and the paper is finished with some concluding remarks.

2. Minimization of peak effect in linear continuous-time systems. A procedure for the minimization of peak effect in a linear continuous-time system with restricted control includes the step, where the upper bound estimation of the free motion of the system should be obtained. For this purpose, the approach based on the calculation of the condition number of the eigenvectors matrix of the system state matrix is used and described in the section. The approach allows us to get the upper bound estimation of the process in dynamics.

Assume a linear continuous-time plant is given in the following form:

$$\dot{x}(t) = Ax(t) + Bu(t); x(0),$$
$$y(t) = Cx(t), \tag{1}$$

where $x \in \mathbb{R}^n, u \in \mathbb{R}^r, y \in \mathbb{R}^m$ are the state vector, the input vector and the output vector, respectively; $x(0) \in \mathbb{R}^n$ is vector of nonzero initial conditions; $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times r}$, $C \in \mathbb{R}^{m\times n}$ are the state matrix, the input matrix and the output matrix of the corresponding dimensions, respectively. It is assumed the pair of matrices (A,B) is controllable matrix pair.

The control law is designed with the pole-placement technique in the form:

$$u(t) = -Kx(t), \tag{2}$$

where matrix $K \in \mathbb{R}^{r\times n}$ consists of the coefficients of the controller provided the required spectrum of eigenvectors and eigenvalues of the state matrix $F = (A - BK)$ of the closed-loop system (1). The feedback matrix K can be calculated with the Sylvester equation:

$$M\Lambda - AM = -BH, K = HM^{-1}, \tag{3}$$

where Λ is $n \times n$ matrix described the desired dynamics of the system, matrix $M = row\{M_i; i = \overline{1,n}\}$ is the invertible square matrix of eigenvectors of matrix F, matrix $H \in \mathbb{R}^{r\times n}$ is chosen such that a pair (Λ, H) is observable. At the same time, the control input u is restricted and satisfies an inequality $-u_{max} \leqslant u \leqslant u_{max}$.

Then, the closed-loop system (1) can be described in the following state-space form:

$$\dot{x} = Fx(t); x(0) = x(t)|_{t=0}, \tag{4}$$

where $F \in \mathbb{R}^{n\times n}$ is a stable state matrix of the closed-loop system with the eigenvalues $\lambda_i < 0, i = \overline{1,n}$. Note, if the original system (4) is unstable, it may not be possible to globally stabilize it under restricted control.

Lemma 1: For the linear system in the form (4) the upper bound estimation $sup\{\|x(t)\|\}$ for the process $\|x(t)\|$ can be given as:

$$\|x(t)\| \leqslant sup\{\|x(t)\|\} = C\{M\}e^{\lambda_M t}\|x(0)\|, \tag{5}$$

where matrix $M = row\{M_i; i = \overline{1,n}\}$ is the invertible square matrix of eigenvectors of matrix F, $C\{M\}$ is the condition number of matrix M [16, 17], λ_M is the maximum eigenvalue of matrix F, defining the degree of stability η of the system (4) according to the expression $\eta = |\lambda_M|$. The norm of initial conditions $x(0)$ is fixed, that $\|x(0)\| = const$.

Proof: The corresponding assertion is proved in [10]. However, for completeness, we give the detailed proof. Assessment of the upper bound of large deviations in the free motion of the continuous-time system (4) is based on the representation of the state matrix F in the following form:

$$M\Lambda = FM, \tag{6}$$

where Λ is a diagonal matrix of eigenvalues, M is a square matrix whose columns are the n linearly independent eigenvectors of F. The solution of equation (4) takes the form:

$$x(t) = e^{Ft}x(0). \tag{7}$$

Using (7) and (6), we get:

$$x(t) = e^{Ft}x(0) = Mdiag\{e^{\lambda_i t}; i = \overline{1,n}\}M^{-1}x(0). \tag{8}$$

Let us form an upper bound for the processes of $x(t)$ in the following form:

$$\begin{aligned} &\|x(t)\| = \|\exp(Ft)x(0)\| = \left\|Mdiag\{e^{\lambda_i t}; i = \overline{1,n}\}M^{-1}x(0)\right\| \leqslant \\ &\|M\| \cdot \left\|diag\{e^{\lambda_i t}; i = \overline{1,n}\}\right\| \cdot \left\|M^{-1}\right\| \cdot \|x(0)\|. \end{aligned} \tag{9}$$

Note, that $C\{M\} = \|M\| \left\|M^{-1}\right\|$ is condition number [18], [16] of the matrix M. Then, expression (9) can be rewritten as:

$$\|x(t)\| \leqslant C\{M\}e^{-\eta t}\|x(0)\|, \tag{10}$$

where η is degree of stability of the system (4) defined as $\eta = \max_i |\lambda_i|; i = \overline{1,n}$.

Therefore, the degree of sufficiency of the upper bound of the process $\|\mathbf{x}(t)\|$ is defined by condition number $C\{M\}$ of the eigenvectors matrix of matrix F.

Remark 1: The upper bound $sup\{\|x(t)\|\}$ of the process $\|x(t)\|$ with minimum sufficiency satisfies the following equations:

$$\|x(t)\| \leqslant sup\{\|x(t)\|\} = C\{\widetilde{M}\}e^{\lambda_M t}\|x(0)\|, \tag{11}$$

where matrix $\widetilde{M}$ is a modified matrix of eigenvectors of matrix F, containing eigenvectors of unity norm [16, 17] in relation to the equality:

$$\widetilde{M} = M \cdot diag\{(\|M_i\|_2)^{-1}; i = \overline{1,n}\}. \tag{12}$$

Remark 2: The upper bound $\|x(t)\|$ of autonomous system process with the initial states $\{\|x(0)\|, t = 0\}$ exceeds its value in $C\{\widetilde{M}\}$ times.

Remark 3: The upper bound is an exponential function. Therefore, the upper bound $sup\{\|x(t)\|\}$ covers the process $\|x(t)\|$, containing deviation at the asymptotic tendency to zero.

Therefore, the peak effect minimization in linear continuous-time systems can be realized with the minimum value of the condition number of the eigenvectors matrix $C\{M\}$. At the same time, the minimum condition number is provided by the assignment of the required structure of eigenvectors that can be described as:

$$C\{M\} = \min_{\xi_i}(C\{row(M_i = \xi_i)\} \& \|\xi_i\| = 1; i = \overline{1,n}). \tag{13}$$

It can be concluded, the peak effect minimization can be provided by the structure of the eigenvectors that delivers the minimum to the condition number of the eigenvectors matrix of the state matrix. The problem (13) is a nondifferentiable optimization problem, that can be solved using one of the nonlinear programming algorithms [19].

3. Minimization of peak effect in linear continuous-time systems with restricted control. In this section, the linear continuous-time system (1) is considered for the case of restricted control and defined as:

$$u(t) = -Kx(t), -u_{max} \leqslant u \leqslant u_{max}. \tag{14}$$

The gramian-based approach is used for the purpose to minimize the peak effect in the system. An appropriate gramian can be obtained on the functional basis of control costs [10, 20]. Thus, for the case of the control law:

$$u(t) = Kx(t) = Ke^{Ft}x(0), \tag{15}$$

for the system (1) can be proved the following Lemma.

Lemma 2: An upper bound and a lower bound of energy consumption [21] for the control (15) of the system (1) can be defined as:

$$\max\|U_\infty\| = \alpha_{\max}^{1/2}\{W_U\}\|x(0)\|, \tag{16}$$

and:

$$\min \|U_\infty\| = \alpha_{\min}^{1/2}\{W_U\}\,\|x(0)\|\,, \tag{17}$$

where $\alpha_{\max}$ and $\alpha_{\min}$ are the maximum and the minimum singular values of the gramian on control costs, respectively. The norm of initial conditions $x(0)$ is fixed, that $\|x(0)\| = const$.

Proof: Let us assume the square of the Euclidean norm of the control vector as energy consumption E_u for the control of a system. Then, if we consider an element $U_t = u_{[0,t)}$ of a linear function space L_T^2, $T = \{t : 0 \leqslant t < \infty\}$, then for the square of the Euclidean norm of the element U_t of the functional space, we can write the following representation:

$$\begin{aligned} E_u = \|U_t\|^2 = \int_0^t u^T(\tau)u(\tau)d\tau = \\ = x^T(0)\int_0^t e^{F^T\tau}K^T K e^{F\tau}d\tau\, x(0), \end{aligned} \tag{18}$$

where $\int_0^t e^{F^T\tau}K^T K e^{F\tau}d\tau \; = W_U(t)$ is called a gramian on control costs or a control costs gramian [20]. The gramian on control costs on infinite time interval satisfies the condition $\lim\limits_{t\to\infty} W_U(t) \; = W_U$ and it is the solution of the matrix Lyapunov equation:

$$F^T W_U + W_U F = -K^T K. \tag{19}$$

Then, if we consider the time function (18) in infinite time interval, we get:

$$\lim_{t\to\infty}\|U_t\|^2 = x^T(0)\lim_{t\to\infty} W_U(t)x(0) = x^T(0)W_U x(0) = \|U_\infty\|^2\,, \tag{20}$$

where $W_U \in R^{n\times n}$ is a solution of the equation (19).

Using singular value decomposition of the control costs gramian W_U we can estimate the upper and lower bounds of control costs on an initial state sphere $x(0)$:

$$\|U_\infty\| = (x^T(0)W_U x(0))^{1/2}, \tag{21}$$

$$\alpha_{\min}^{1/2}\{W_U\}\,\|x(0)\| \leqslant \|U_\infty\| \leqslant \alpha_{\max}^{1/2}\{W_U\}\,\|x(0)\|\,. \tag{22}$$

The Lemma is proved.

That means the gramian on control costs is an estimation of energy costs for the control law realization. The estimation of the control costs allows us to find the optimal pole placement for a closed-loop researched system using the upper bound (16) for the control cost minimization. The restriction to the control $-u_{max} \leqslant u \leqslant u_{max}$ should be taken into account together with the upper bound (16) of the control cost. This allows us to take the following approach.

Approach to the peak effect minimization. The peak effect minimization for linear continuous-time systems with restricted control (1), (14) can be realized with the minimization of the condition number of the eigenvector matrix and the minimization of the upper bound of the control costs.

The approach to the peak effect minimization for linear continuous-time systems with restricted control is realised on the consideration of two indexes together, that are the upper bound of the control cost $\alpha_{\max}^{1/2}\{W_U\}$ and the condition number of the eigenvector matrix $C\{M\}$. That indexes can be aggregated together due to the property of the condition number changing within the interval $1 \leqslant C\{*\} < \infty$. Then, the aggregated index takes the form:

$$J(C,U) = \alpha_{\max}^{1/2}\{W_U\}C\{M\}. \tag{23}$$

The index $J(C,U)$ takes into account as condition number of the eigenvector matrix as the control costs. Then, it is reasonable to use the index as a mathematical tool for the peak effect minimization. It should be noted the functional $J(C,U)$ is not convex and may have several minimums (local minimums).

4. Procedure for the peak effect minimization. The procedure for peak effect minimization in linear continuous-time systems with restricted control can be realized according to the following steps:

1) Define a continuous-time system in the form (1);

2) Form an aggregated index $J(C,U)$ in the form (23):

• specify initial eigenvalues $\Lambda_0 = diag\{\lambda_{0i} \in [\lambda_{min}, 0); i = \overline{1,n}\}$ of the state matrix F of a closed-loop system (4), where λ_{min} provides the required stability degree of the system;

• calculate a feedback matrix K with the Sylvester equation (3);

• calculate the state matrix $F = (A - BK)$ of a closed-loop system;

• calculate the condition number $C\{M\}$ of eigenvectors matrix and the gramian of control costs (19) to get the index $J(C,U)$;

3) Find the minimum of above the specified index $J(C,U)$ subject to $\Lambda = diag\{\lambda_i \in [\lambda_{min}, 0); i = \overline{1,n}\}$ using one of nonlinear programming algorithms [19, 22];

4) Fix the results in the form of the coefficients k_i of the feedback matrix K, that provide the minimum of peak effect in the free motion of the system with control saturation;

5) Simulate the system and analyze the performances.

The combination of methods provides the peak effect minimization in the free motion of continuous-time systems with restricted control.

5. Example. As an example, the trajectory of the relative motion of two satellites moving in a circumcircle orbit in the central gravitational field of the Earth is considered [23]. The design is based on the linearized dynamic model of the satellite system. The behavior of the system with restricted control is considered.

The example section consists of two subsections. The first subsection describes the behaviour of the system for two cases: 1) the input saturation is not taken into account; 2) the input saturation is taken into account at the control design stage. It is illustrated, that limitation on control has a significant impact on the system's behavior and can lead the system to stability loss. The minimization problem of the peak effect for the satellites relative trajectories with the proposed procedure is considered in the second subsection. The simulation results demonstrate the efficiency of the procedure.

5.1. Modelling of two satellites system. The linearized equations of the relative motion of satellites are given in the state space form (1), where:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -2\omega \\ 0 & 0 & 0 & 1 \\ 0 & 2\omega & 3\omega^2 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, u = -\varphi(y).$$

Here, $x = \begin{bmatrix} x_{12} & \dot{x}_{12} & z_{12} & \dot{z}_{12} \end{bmatrix}$ is the system state vector, where x_{12} denotes the difference in coordinates of the second and the first satellites, $x_{12} = x_2 - x_1$, $z_{12} = z_2 - z_1$. The control input u is described by the saturation function $\varphi(y) = sat(u_{max})$ and satisfies an inequality $-u_{max} \leqslant u \leqslant u_{max}$, $u_{max} = 2.4 \times 10^{-5} m/s^2$.

The averaged angular velocity ω of the satellites in orbit satisfies the expression $\omega = \sqrt{\mu/a^3}$, where $\mu = GM$, G is the gravitational constant, M is the mass of the central body, and a is the semimajor axis of the satellite's orbit. In this case, for the Earth, $\mu = 398.603 \times 10^9 m^3 s^{-2}$, $\omega = 0.001172$.

The control law is designed with the pole-placement technique in the form (2). The coefficients of the controller $K = \begin{bmatrix} k_1 & k_2 & k_3 & k_4 \end{bmatrix}$ is assigned to provide the required spectrum of eigenvalues for the matrix of the closed-loop system $F = A - BK$. Note, the condition number of the eigenvectors matrix of the states matrix A is $C(A) = 1.49 \times 10^{19}$.

Consider the behavior of the satellites relative trajectories, when there is no restriction on control and control is designed in the form (2). The coefficients of the controller k_i are defined to minimize the condition number of the eigenvector matrix (13), which are $K = \begin{bmatrix} -0.0001 & 0.1265 & 0,0024 & 1.2165 \end{bmatrix}$.

The simulation results for the satellite's coordinates x_{12} and z_{12} with the defined initial conditions $x_{12}(0) = 200m$, $\dot{x}_{12}(0) = 0.025m/s$, $z_{12}(0) = -50m$, $\dot{z}_{12}(0) = -0.025m/s$ and the satellites relative trajectories on the (x,z) plane are depicted in Figures 1 and 2, respectively. The simulation time is confined to $t_{fin} = 54000s = 15h$. Let us consider regulation time t_r when the relative trajectory on the (x,z) plane reaches the circle with given radius R and does not leave it then, therefore $t_r = \max_t(\sqrt{x_{12}^2 + z_{12}^2} > R)$ [23]. It is assumed that collision is escaped within this circle.

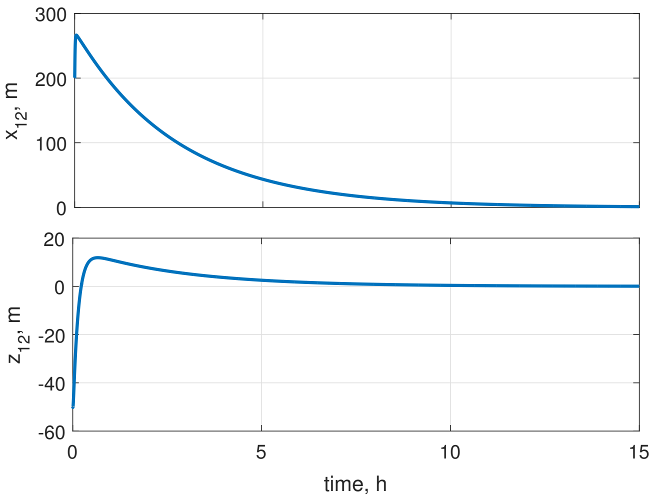

Fig. 1. Satellites relative trajectories $x_{12}(t)$ and $z_{12}(t)$ with nonzero initial conditions. The control $u(t)$ is not restricted

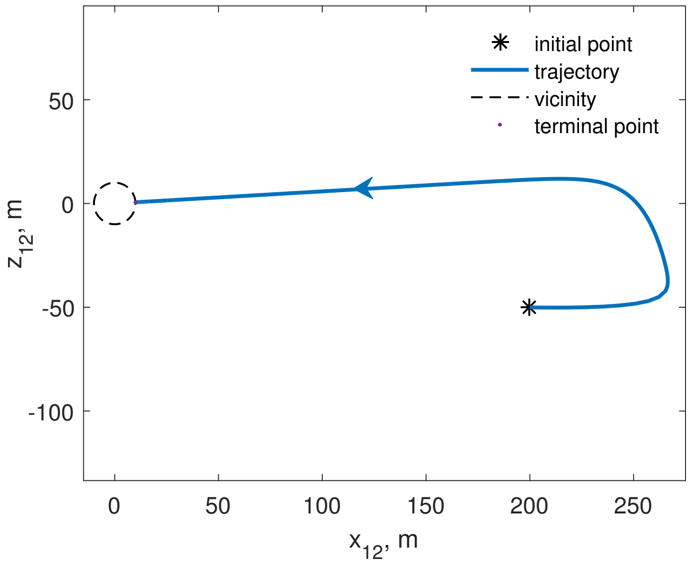

Fig. 2. Satellites relative trajectories on the (x, z) plane with nonzero initial conditions. The control $u(t)$ is not restricted

It can be observed that the peak effect exists in the stable satellites relative trajectories $x_{12}(t)$ and $z_{12}(t)$. And it is about 270 m for the trajectories $x_{12}(t)$, and it is about 17 m for the trajectories $z_{12}(t)$. Here, the regulation time $t_r = 33148s = 9.2h$, and a minimum of the condition number $C(M) = 1467$ of the eigenvectors matrix was found for $\Lambda = \{-0.0996 \quad -0.0001 \quad -0.0248 \quad -0.002\}$.

For the case of restricted control $|u| \leqslant u_{\max}$ with the same feedback coefficients k_i the situation is changed. The closed-loop system becomes unstable. The behavior of the satellites relative trajectories takes the curves represented in Figures 3 and 4, respectively. Therefore, limitation on control has a significant impact on the system's behavior and can lead the system to stability loss.

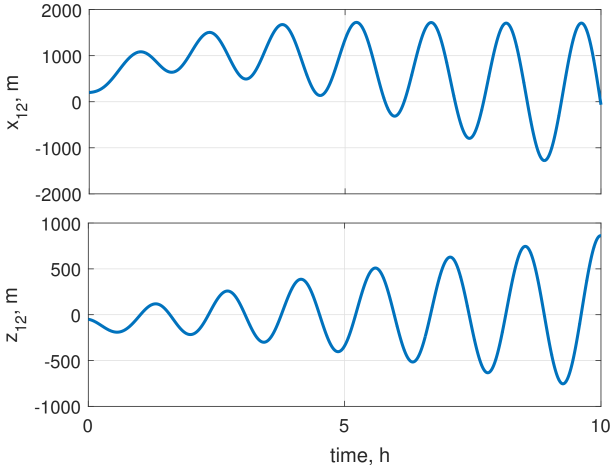

Fig. 3. Satellites relative trajectories $x_{12}(t)$ and $z_{12}(t)$ with nonzero initial conditions and the restricted control $|u| \leqslant u_{\max}$

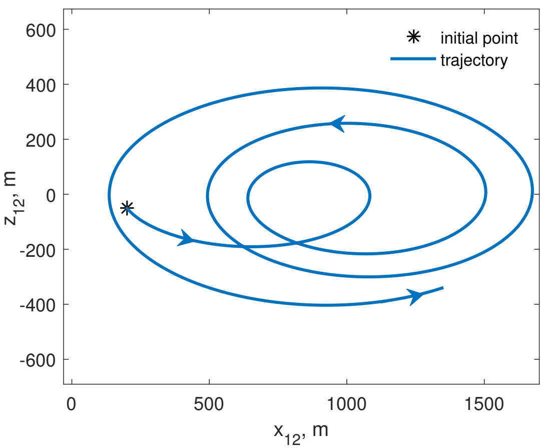

Fig. 4. Satellites relative trajectories on the (x,z) plane with nonzero initial conditions and the restricted control $|u| \leqslant u_{\max}$

5.2. Minimization of peak effect for the satellites relative trajectories. In this section, the proposed iterative procedure for the peak effect minimization is applied to the satellites system. It should be noted, an interval for the eigenvalues search is limited by the channels capacity [23] and the range of admissible algebraic spectrum of eigenvalues is varied within the interval $\lambda_i \in [-1, 0)$.

First of all, the case of minimization of control costs is considered. The coefficients of the controller k_i are defined to provide the condition (16), that are $K = \begin{bmatrix} -9.12 \cdot 10^{-9} & 0.0018 & 3.24 \cdot 10^{-6} & -8.6 \cdot 10^{-5} \end{bmatrix}$.

The simulation results for the satellite's coordinates x_{12} and z_{12} with above defined initial conditions and the satellites relative trajectories on the (x, z) plane are depicted in Figures 5 and 6, respectively. Here, the regulation time $t_r = 31810s = 8.83h$, and a minimum of the gramian on control costs $W_u = 0.0317$ was found for $\Lambda = \{-0.0004 \quad -0.0004 \quad -0.0004 \quad -0.0004\}$.

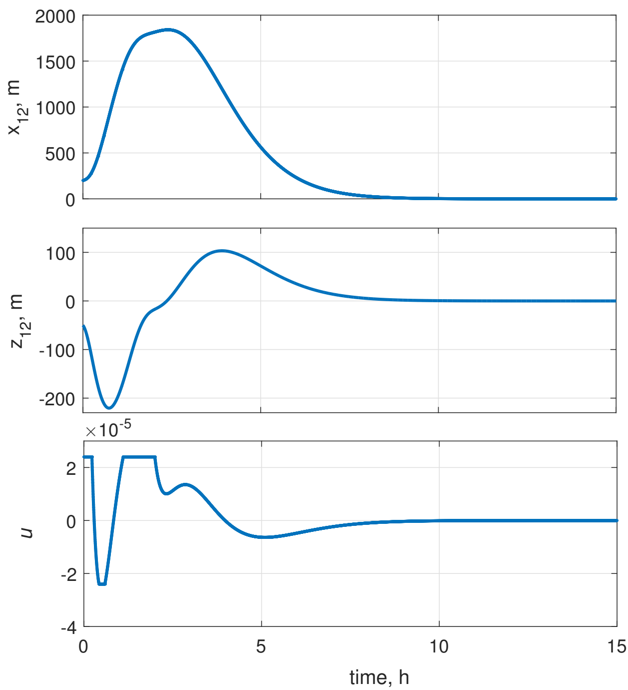

Fig. 5. Satellites relative trajectories $x_{12}(t)$ and $z_{12}(t)$ with nonzero initial conditions and the restricted control $|u| \leqslant u_{\max}$. Minimization of peak effect by the minimization of control costs

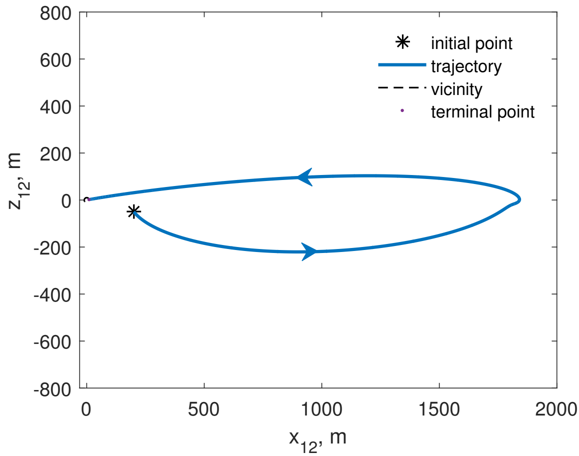

Fig. 6. Satellites relative trajectories on the (x,z) plane with nonzero initial conditions and the restricted control $|u| \leqslant u_{\max}$. Minimization of peak effect by the minimization of control costs

It can be seen, the researched system is stable and has a peak effect of about 1800 m for the satellite's coordinate x_{12} and it is about 220 m for the satellite's coordinate z_{12}.

Then, the case of peak effect minimization with the aggregated index $J(C,U)$ in the form (23) is considered and minimization of the condition number of eigenvectors matrix together with the singular value of the gramian on the control costs is realized by the controller $K = \begin{bmatrix} -9.65 \cdot 10^{-8} & 0.006 & 1.28 \cdot 10^{-5} & 0.004 \end{bmatrix}$.

The simulation results for the satellite's coordinates x_{12} and z_{12} with above defined initial conditions and the satellites relative trajectories on the (x,z) plane are depicted in Figures 7 and 8, respectively. Here, the regulation time $t_r = 57765s = 16h$ and the minimum of the aggregated index $J(C,U) = 804.11$ were found for $\Lambda = \{-0.0006 \quad -0.0033 \quad -0.0021 \quad -0.0001\}$. Obviously, the peak effect is minimized and reaches 1300m for the satellite's coordinate x_{12} and 190m for the satellite's coordinate z_{12}. But it should be noted, the peak effect minimization leads to the regulation time changing. So, the regulation time, in this case, was increased almost twice.

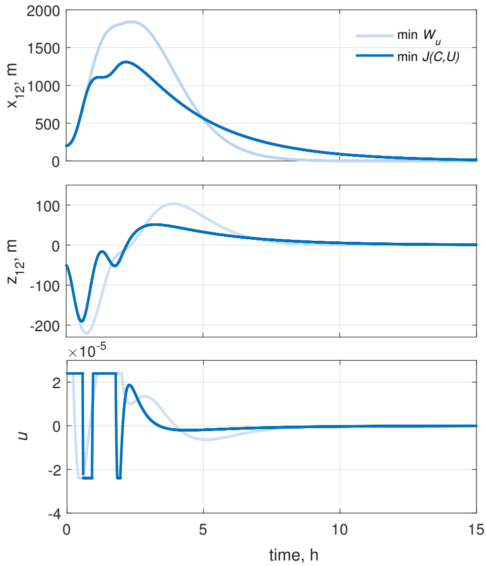

Fig. 7. Satellites relative trajectories $x_{12}(t)$ and $z_{12}(t)$ with nonzero initial conditions and the restricted control $|u| \leqslant u_{\max}$. Minimization of peak effect with the proposed procedure

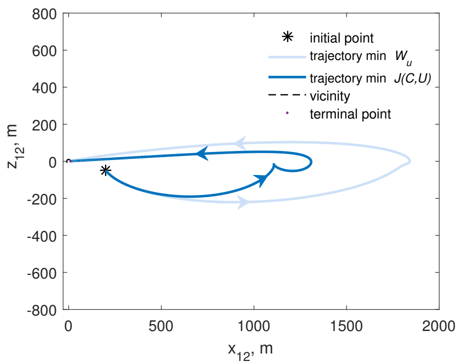

Fig. 8. Satellites relative trajectories on the (x, z) plane with nonzero initial conditions and the restricted control $|u| \leqslant u_{\max}$. Minimization of peak effect with the proposed procedure

Figure 9 illustrates the norm of the state vector $\|x(t)\|$ and the restricted control u for different values of the controller coefficients K_i. The plots are presented for the cases of eigenvalues deviations from the proposed values Λ for 20% in one direction and in the other one. It can be seen, that the proposed controller allows us to find an optimal compromise between the control costs and the peak effect in the trajectories of the free movement of satellites.

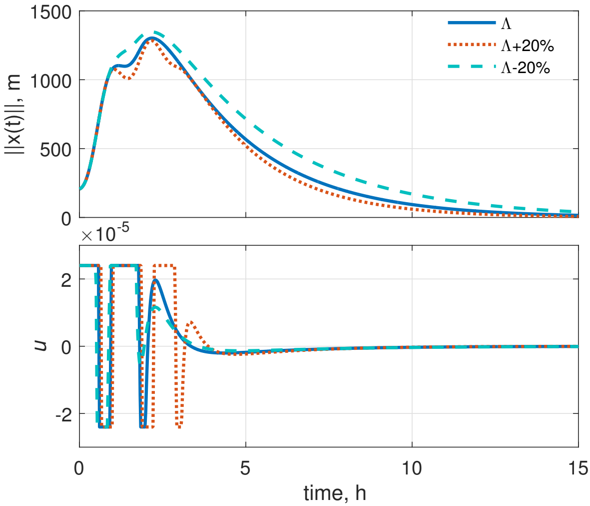

Fig. 9. Plots of the state vector $\|x(t)\|$ and the restricted control u for different values of the controller coefficients K_i

6. Conclusion. The aim of the paper was to minimize the peak effect in stabilization systems under any non-zero initial conditions with restricted control. The iterative procedure was suggested for the peak effect minimization problem. The procedure was based on a combination of the recently proposed gramian-based approach and the theory of using the condition number of the eigenvector matrix for the upper bound estimations of the system processes. It was established the correct structure of eigenvectors that delivers the minimum value to the condition number of the eigenvector matrix of the closed-loop system should be considered together with the maximum singular value of a gramian on control costs to provide the peak effect minimization to the system's behaviour.

The procedure was applied to the system of two satellites. Minimization of peak effect for the satellites relative trajectories was reached. The simulation results demonstrated the efficiency of the procedure.

As future work, it is supposed to consider the case of the system with restricted control and input additive uncertainties to study the peak effect in the system.

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DOI 10.15622/ia.22.3.6

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МИНИМИЗАЦИЯ ОТКЛОНЕНИЙ В ТРАЕКТОРИЯХ СВОБОДНОГО ДВИЖЕНИЯ ЛИНЕЙНЫХ СИСТЕМ С ОГРАНИЧЕНИЯМИ ПО УПРАВЛЕНИЮ

Дударенко Н.А., Вундер Н.А., Мельников В.Г., Жиленков А.А. **Минимизация отклонений в траекториях свободного движения линейных систем с ограничениями по управлению.**

Аннотация. Рассматривается задача минимизации отклонений в траекториях свободного движения линейных систем с ограничениями по управлению. Предложен итеративный алгоритм для минимизации отклонений с использованием технологии системных грамианов и числа обусловленности матрицы собственных векторов устойчивой системы. Минимизация затрат на управление базируется на анализе сингулярного разложения грамиана затрат на управление с последующим формированием мажорантных и минорантных грамианных оценок. Минимизация отклонений в траекториях свободного движения систем осуществляется путем минимизации числа обусловленности матрицы собственных векторов матрицы состояния замкнутой системы, при этом матрица состояния с желаемыми спектрами собственных чисел и собственных векторов конструируется на основе обобщенного модального управления. В основе разработки итеративного алгоритма для минимизации отклонений в траекториях движения линейных систем при ненулевых начальных условиях с ограничениями по управлению лежит агрегированный показатель, позволяющий сформировать систему с минимальными отклонениями в траекториях ее свободного движения при минимальных затратах на управление. Данный показатель учитывает одновременно как оценку грамиана затрат на управление, так и число обусловленности матрицы собственных векторов устойчивой замкнутой системы. Минимизация агрегированного показателя позволяет обеспечить минимальные отклонения в траекториях свободного движения систем рассматриваемого класса. Алгоритм апробирован на примере системы с ограниченным входом, описывающей относительное движение двух спутников. Рассмотрено два случая минимизации отклонений. В первом случае минимизация отклонений в траекториях свободного движения спутников выполнена только за счет минимизации грамиана затрат на управление. Во втором случае минимизация отклонений осуществлена с применением разработанного алгоритма. Полученные результаты иллюстрируют эффективность предложенного алгоритма и уменьшение величины отклонений в траекториях относительного движения спутников.

Ключевые слова: число обусловленности, затраты на управление, ограничение по входу, свободное движение, грамиан, эффект всплеска, спутники, оценка сверху.

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