

COMPOUND MODEL OF FADING

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Introduction: Mathematical model of a signal at the output of a radio channel is the most important stage in constructing of the channel simulators using for testing of a communication system performance. The signal at the output of propagation channel suffers from random fading and Doppler frequency spread. Correlation function of the fading signal envelope is exponential, while its probability density is supposed usually to be Rayleigh. As a result, the quadrature representation of the fading signal is relevant. This approach is not correct in certain scenarios, particularly in mobile communication applications, for example in the case of shadowing. **Purpose:** To find a rather universal method of the fading signal modelling which takes into account an additional requirement of obtaining typical distribution of random Doppler spread of the received signal carrier frequency depending of the mobile speed. **Results:** The effectiveness of the presented compound method was tested by generating of a signal with sub-Rayleigh envelope distribution (*m*-Nakagami with *m* = 0,7) and with the desired spectral form. It may be useful for simulating of mobile channel propagation channel with arbitrary distributed fading, while preserving the Doppler spectrum spread characteristics. In the framework of the presented concept the spectrum shape is formed via linear filtering of wide-band Gaussian random process. The resultant complex process having Rayleigh distributed envelope is multiplied by a positive real-valued stochastic process with exponential correlation function, providing opportunity to generate narrowband processes with any envelope distribution, while preserving the classical Doppler spectrum shape as defined by a system parameters.

Keywords – Multipath Fading, Mobile Radio Propagation Channel, Compound Stochastic Processes.

Introduction

The propagation medium of mobile wireless communication is characterized by two phenomena: multipath propagation and signal fading. In the simulation of the land mobile radio fading signal the most widely used baseband representation is a quadrature baseband scheme [1] deploying filtering of a Gaussian white noise with identical spectrum shaping filters of the in-phase and quadrature components. The spectrum shape of the fading radio signal is characterized by a spread of Doppler frequency which has a random cosine distribution while its typical power spectrum density $S(f)$ for particular mobile speed, antenna and polarization is given [2, 3] by

$$\begin{cases} S(f) = \frac{E^2}{2\pi f_m} \left[1 - (f/f_m)^2 \right]^{-1/2}, & f < f_m, \\ S(f) = 0, & f \geq f_m \end{cases}, \quad (1)$$

where E is a *rms* value of the signal envelope and f_m is the maximum Doppler spread with respect to mobile speed and the transmitted signal carrier frequency. For computer simulations an all-pole filter of the form $H(z) = k/P_n(z)$ can be used, where k is a gain constant and $P_n(z) = a_0 + a_1 z^{-1} + \dots + a_n z^{-n}$. The coefficients $\{a_i\}_{i=0}^n$ are to be derived with respect to the mobile speed, carrier frequency and other physical parameters. The known propagation channel simulators generate a complex signal $x_n(t)$ having Rayleigh distributed envelope. However, this distribution is not always

the best model of the fading signal [4, 5]. We suggest here a compound generating model of a fading as narrow-band process with non-Rayleigh distributed envelope but with the same power spectrum density.

Signal Generation Method

Multiplying the output $x_n(t)$ of a classical channel simulator by a positive realvalued stochastic process $s(t)$, we obtain so called compound stochastic process

$$y(t) = x_n(t) \cdot s(t) \quad (2)$$

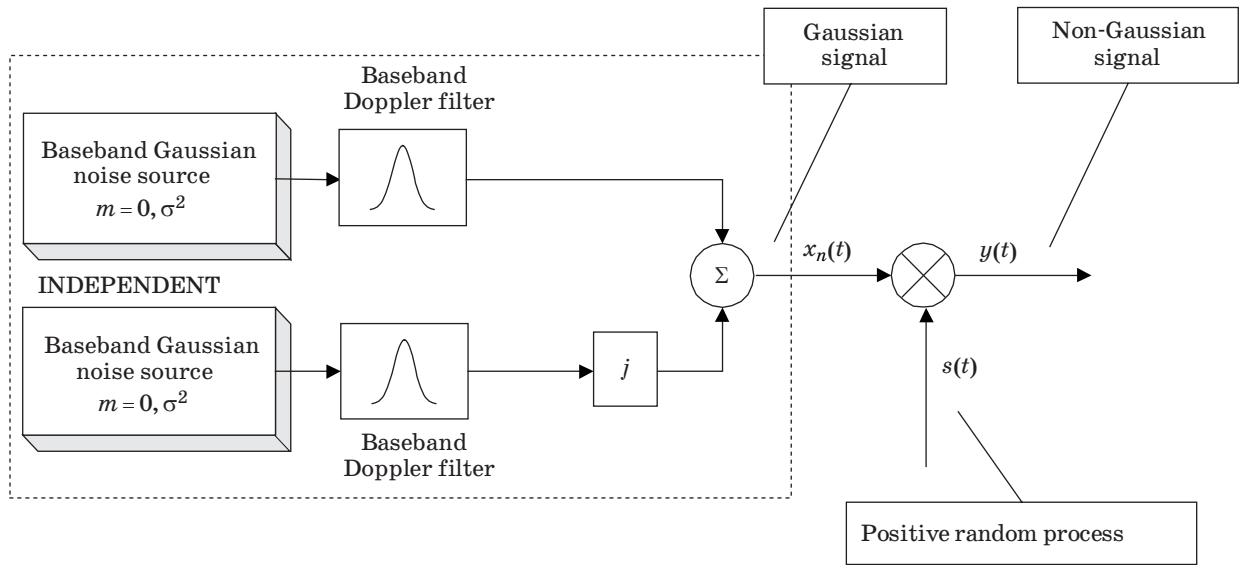
with modified envelope distribution. The Gaussian narrowband process $x_n(t)$ in (2) defines the spectral (correlation) properties of $y(t)$, and the modulation $s(t)$ defines its distribution. The block diagram of the baseband fading simulator is shown on Fig. 1.

Let the envelope of the resultant process $y(t)$ is the generalized Gamma distribution [6]

$$f_{A_y}(A_y) = \frac{\gamma \beta^{\alpha/\gamma} A_y^{\alpha-1}}{2\Gamma(\alpha/\gamma)} \exp(-\beta A_y^\gamma), \quad (3)$$

which includes as particular cases the Rayleigh Probability Density Function (PDF) ($\alpha = 2, \gamma = 2$), the Nakagami PDF ($\alpha = 2m, \gamma = 2$) and the Weibull PDF ($\alpha = \gamma$) [7]. The linkage between $f_{A_y}(A_y)$ and the distribution $f(s)$ of the modulating process is well known

$$f_{A_y}(A_y) = \int_0^\infty f(A_y/s) f(s) ds, \quad (4)$$



■ Fig. 1. Baseband fading simulator block diagram

where the process $s(t)$ has a one-sided distribution and its mean value is to be unity to keep the mean energy of the process $y(t)$ the same as the mean energy of the Gaussian process $x_n(t)$. This leads to the following condition

$$\int_0^{\infty} s f_s(s) ds = 1, \quad (5)$$

which is to be taken into account when we are looking for $f_s(s)$ as a solution of the integral equation with the left side defined by (4). Since the modulating process $s(t)$ has certain correlation function, it will affect the spectral characteristic of the resultant signal.

When $s(t)$ is the so called λ -process [8], which has an exponential correlation function with decay λ , it is possible to compensate the above mentioned phenomena. The stochastic differential equation generating the λ -process $s(t)$ with the PDF $f_s(s)$ is written as [9]

$$\dot{s} = -\lambda s + \sqrt{\frac{\lambda}{f_s(s)} \int_{-\infty}^s s f_s(s) ds} \xi(t), \quad (6)$$

where $\xi(t)$ is the White Gaussian noise with unit spectral density.

Since the power spectrum of a λ -process is formed by a single pole transfer function, where the pole is related to the correlation interval, one shall correct the poles of the transfer function of the spectral shaping filters. Let us consider the characteristic polynomial of the spectral shaping filter

$$P_n(z) = a_0 + a_1 z^{-1} + \dots + a_n z^{-n} \quad (7)$$

and let the poles $\{z_{ni}\}_1^N$ represent the roots of this polynomial. The Nyquist samples of $s(t)$ are written as

$$s_i = \alpha_i + j\omega_i = F_s \ln z_i, \quad i = \{1, N\}, \quad (8)$$

where F_s is the sampling frequency. The correlation interval of the desired λ -process is to be defined in such a way that the condition

$$\lambda = \max\{a_i\}_1^N / 2 \quad (9)$$

holds.

Now we can correct the poles of the shaping filter

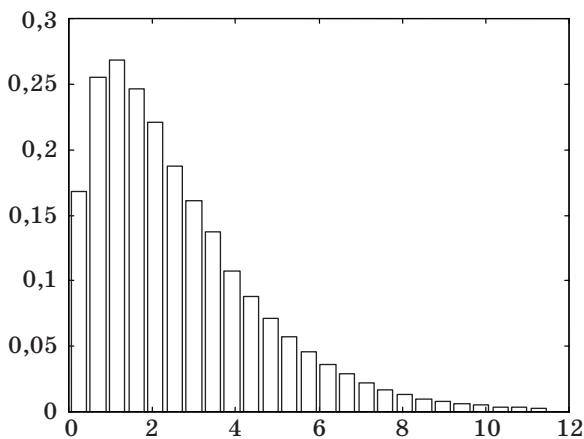
$$S_{ni} = s_i + \lambda = \alpha_i + \lambda + j\omega_i; \quad (10)$$

$$z_{ni} = z_i \exp\left(\frac{\lambda}{F_s}\right).$$

This will ensure that the modified form filter is stable (if the original was a stable one). Furthermore, if Gaussian process $x_n(t)$ has a spectral characteristic defined by the shaping filter with the roots $\{z_{ni}\}_1^N$ as defined in (9) and the modulating random process $s(t)$ is a process with decay of correlation function λ defined by (8), then the resultant compound process $y(t)$ has the same spectrum as the original Gaussian process $x_n(t)$.

Results

The described scheme was used for simulation of a propagation channel for the case of the mobile velocity 45 mph, carrier frequency 900 MHz and sam-



■ Fig. 2. Envelope distribution of the compound process

pling rate of 36 kbps. The multiplying random process was distributed according to Nakagami distribution law [7] with parameter $m = 0,7$ (the sub-Rayleigh fading), corresponding to (4) with parameters $1 \leq \alpha < 2$, $\gamma = 2$. The envelope probability density function of the compound process is shown on Fig. 2.

While the distributions of the envelopes for two processes are quite different, the results show a very good matching of correlation functions. Thus the presented method provides a procedure for generating of non-Gaussian processes having the desired spectral form. It may be useful for simulating of radio channel propagation with arbitrary distributed fading, while preserving the Doppler spectrum spread characteristics.

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Компаундная модель фединга

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Введение: выбор математической модели радиосигнала является наиболее важным этапом при конструировании имитаторов, используемых для исследования эффективности мобильных систем связи. Для такого сигнала характерны специфическая форма доплеровского спектра и случайные экспоненциально коррелированные изменения амплитуды (фединг). Обычно предполагается, что фединг имеет распределение Релея, которому соответствует представление с помощью гауссовых квадратурных компонент. Тем не менее для канала мобильной радиосвязи зачастую характерен фединг с распределением, отличным от релеевского (например, в отсутствие прямой видимости). **Цель исследования:** создание компаундной модели сигнала с любым распределением фединга с типичным распределением доплеровского смещения, зависящего от несущей частоты и скорости движения передатчика. **Результаты:** эффективность компаундной модели фединга была протестирована при моделировании канала подвижной связи с федингом, более глубоким, чем релеевский (распределение Накагами с параметром $m = 0,7$), и со спектром, определяемым несущей частотой и скоростью движения передатчика. В рамках предлагаемого подхода форма спектра затухающего сигнала формируется с помощью линейной фильтрации широкополосного нормально распределенного процесса. Выход фильтра умножается на процесс с экспоненциальной корреляционной функцией и с определенным односторонним распределением. Результирующий узкополосный процесс обладает заданным распределением огибающей и типовым доплеровским спектром, определяемым параметрами системы.

Ключевые слова — многолучевое затухание, канал распространения мобильной радиосвязи, смешанные стохастические процессы.